

## Ternary Systems for the Liquid Immiscibility Gap Technology

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(Received 17 May 2013; published online 30 August 2013)

The computer models for T-x-y diagrams with the mono- and invariant monotectic equilibrium, and monovariant syntectic equilibrium by means of kinematical simulation of phase regions boundaries are considered. The effect of syntectic equilibrium changing for monotectic one is investigated with the material balances visualization.

**Keywords:** Immiscibility Gap, Phase Diagram, Computer Model.

PACS numbers: 64.70.kd, 81.30.Bx

### 1. INTRODUCTION

T-x-y diagrams with liquid immiscibility is employed widely for the solution of problem associated with a refractory compounds obtaining (borides, silicides and intermetallides). Besides the well-known flax method with only one solvent, the immiscibility gap method for the metallic systems was offered in the third millennium [1-4]. In this method the synthesis and crystallization of compounds have place on the border of two solvents. As this technology includes many unknown details, computer models of phase diagrams will be a useful tool for the understanding.

Phase diagrams with immiscibility can be calculated and investigated using both thermodynamic methods [5-11] and mathematic ones. The elaboration of the algorithms and attendant software, based on a mathematical description of phase diagrams surfaces, permit to construct the space models of T-x-y diagrams and to investigate them. In this case the set of initial experimental data concerning the type of interaction in the system and the data about surfaces contours are used. The hypothetic data can be applied at the deficiency of such initial information. It allows to solve a problem of phase diagram visualization as the three-dimensional object, and to avoid its fragmentary representation [12-14].

The kinematical method of T-x-y diagrams surfaces description is chosen as the most efficient method [15-16]. Here, the kinematical surface is given by the motion of the forming curve along to the set of directing curves, described by the interpolation polynomials. Such method constructs the surfaces with complex geometrical structure [17], involving the folds, extrema, cuts, saddle points.

Computer templates is convenient to use for the simulation of phase diagrams with immiscibility surfaces, having a great quantity of topological modifications, produced by different arrangements and intersections of monovariant lines and immiscibility surfaces [18-19], and at the degeneration of solidus and solvus surfaces. They can be degenerated either on the prism edge, or on the prism side [20].

### 2. T-X-Y DIAGRAMS COMPUTER MODELS

Let's consider the models of T-x-y diagrams with mono- and invariant monotectic equilibria and monovariant syntectic equilibrium as an example. The coordinates of binary and ternary points on the surfaces contours are given an initial data at the phase diagrams simulation. The following designation are used:  $i$  – immiscibility surface,  $Q$  – liquidus,  $S$  – solidus,  $V$  – solvus,  $i^r$  – ruled surfaces formed by the intersection lines of immiscibility surface with liquidus on the one hand and the solidus surface folds or intersection lines of two solidus surfaces on the other hand,  $Q^r$  – formed by the monovariant lines of liquidus and the intersection lines of solidus surfaces with the solvus ones,  $S^r$  – formed by the intersection lines of solidus surfaces with the solvus ones,  $V^r$  – formed by the intersection lines of the solvus surfaces,  $H$  – horizontal planes.

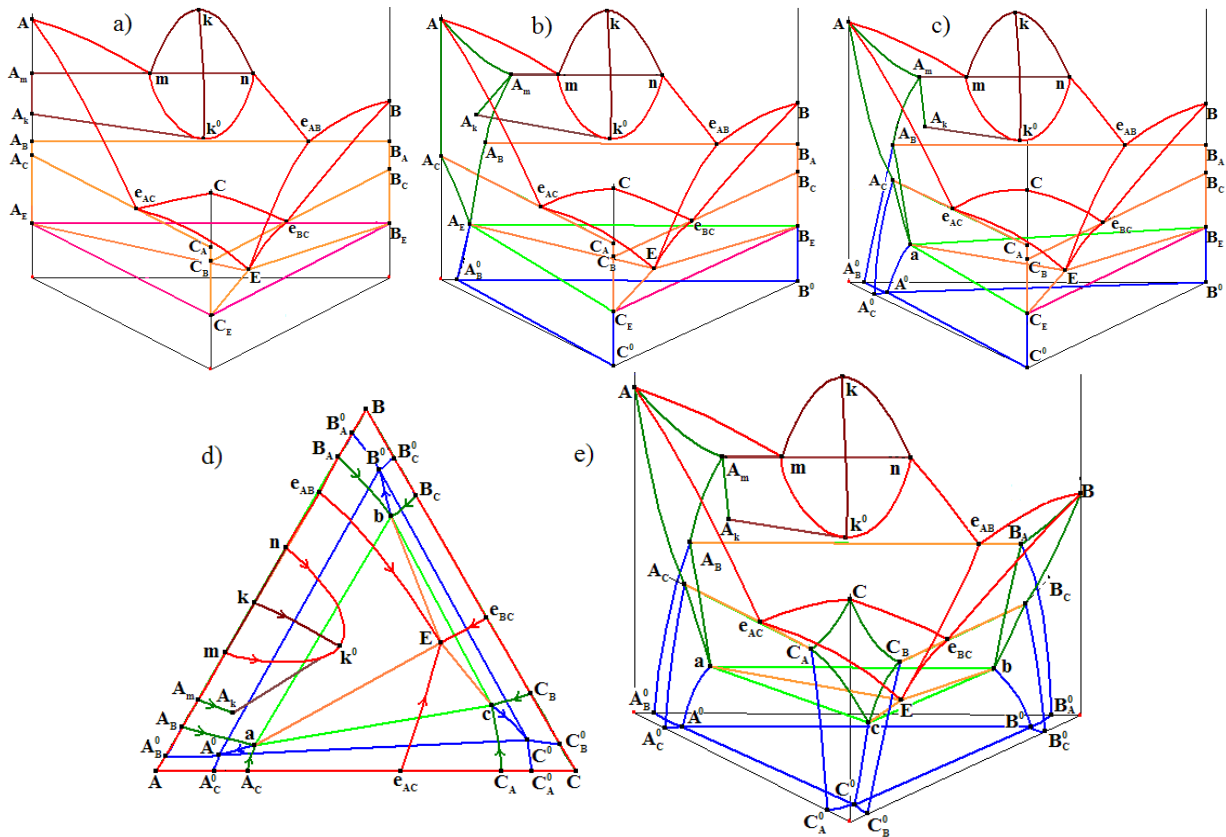
#### 2.1 T-x-y diagram with monovariant monotectic equilibrium

Considered phase diagram without solid solubility is formed by a surface of liquid immiscibility ( $i$ :  $mknk^0$ ), 3 liquidus surfaces ( $Q_A$ :  $Amk^0n_{eAB}E_{eAC}$ ,  $Q_B$ :  $Be_{AB}E_{eBC}$ ,  $Q_C$ :  $Ce_{AC}E_{eBC}$ ), 9 ruled surfaces ( $i^r$ :  $mknk^0$ ,  $i^r_m$ :  $A_mnk^0A_{ko}$ ,  $i^r_n$ :  $A_mnk^0A_{ko}$ ,  $Q^{r_{AB}}$ :  $ABe_{AB}E_{eAE}$ ,  $Q^{r_{BA}}$ :  $BAe_{AB}E_{eBE}$ ,  $Q^{r_{AC}}$ :  $ACE_{AC}E_{eAE}$ ,  $Q^{r_{CA}}$ :  $CAe_{AC}E_{eCE}$ ,  $Q^{r_{BC}}$ :  $Bce_{BC}E_{eBE}$ ,  $Q^{r_{CB}}$ :  $CBe_{BC}E_{eCE}$ ) and a horizontal complex at invariant eutectic point E ( $H_E$ :  $Ae_{BE}Ce_{EE}$ ), consisted of 4 simplexes (Fig. 1a) [16].

The coordinates of initial components (A, B, C), binary eutectic points ( $e_{AB}$ ,  $e_{AC}$ ,  $e_{BC}$ ), a ternary eutectic point (E), points on the contour of immiscibility cupola ( $k$ ,  $k^0$ ,  $m$ ,  $n$ ) and appropriate points on the edges of prism ( $I_{eLL}$ ,  $I_E$ ,  $I_{m(n)}$ ,  $I_{k0}$  ( $I=A,B,C$ )) are taken as the initial data for the phase diagram assemblage. By the use of given computer model we can consider step by step the appearance of solid solubility at one (Fig. 1b-c), two and three components (Fig. 1d-e, Table 1). Fig. 1b-c presents the appearance of solidus and solvus surfaces in component A in binary system AB (Fig. 1b) and both

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**Fig. 1** – Model of phase diagram with monovariant monotectic equilibrium without (a) and with solid solubility in one (b-c) and three (d-e) components (additional information in Table 1)

binary system A-B and A-C (Fig. 1c). In first case we can see the line of solidus ( $AA_m, A_mAB, ABAE, ACAE, A_mAk$  - pseudo-folds) and solvus ( $A_EA^0B$ ), but second case visualizes the surfaces of solidus ( $AAcA_BA_m$  with fold  $A_mAk$ ) and solvus ( $ABaa^0A^0B, Acaa^0A^0C$ ).

Simulation of solidus and solvus surfaces at all three compounds gives the complete T-x-y diagram with solid phases (Fig. 1d-e). As models come to be more complex, the coordinates of points on the contour of solidus and solvus surfaces are added as initial data of model.

**Table 1** – Surfaces within phase diagram with monovariant monotectic equilibrium (view and number) for the different variants of solidus and solvus degeneration (Fig. 1)

Surfaces	a	b	c	d-e
immiscibility surface (i)	1	1	1	1
liquidus surfaces (Q)	3	3	3	3
solidus surfaces (S)	-	4 line in AB binary system	1	3
solvus surfaces (V)	-	1 line in AB binary system	1	6
ruled surfaces of i ( $i^r$ )	3	3	3	3
ruled surfaces of Q ( $Q^r$ )	6	6	6	6
ruled surfaces of S ( $S^r$ )	-	1	2	3
ruled surfaces of V ( $V^r$ )	-	1	2	3
horizontal complex (H)	1	1	1	1
Total	14	16	20	29

### 2.2 T-x-y diagram with invariant monotectic equilibrium

T-x-y diagram (Fig. 2, Table 2) with solid phase solubility includes a surface of liquid immiscibility (i:  $mMk^0Qnk$ ), 4 liquidus surfaces ( $Q_A: AmMeAC, Q_{A1}: neABEQ, Q_B: BeABEeBC, Q_C: CeACeEBc$ ), 4 solidus surfaces ( $S_A: AmBAqAC, S_{A1}: mBAbAEaq, S_B: BbABeBc, S_C: CcACqCEcb$ ), 6 solvus surfaces ( $V_{AB}: abAeA^0Ea^0B, V_{AC}: acaqAeA^0Ea^0C, V_{BA}: babEb^0Eb^0A, V_{BC}: bcbEb^0Eb^0C, V_{CA}: cacqCEC^0Ec^0A, V_{CV}: cbCEC^0Ec^0B$ ), 21 ruled surfaces ( $6i^r+8Q^r+4S^r+3V^r$ ) and 2 horizontal complexes at invariant points E and M(Q). The coordinates of initial components (A, B, C), the binary eutectics ( $e_{AB}, e_{AC}, e_{BC}$ ), ternary points (E, M, Q), the points of the liquid immiscibility cupola (m, n, k,  $k^0$ ), the binary and ternary points of the surfaces of the solidus ( $m_B, ab, ac, aE, aq; b_A, bc, bE; c_A, cb, ce, cQ, ck_0$ ) and the solvus ( $a^0B, a^0C, a^0E; b^0A, b^0C, b^0E; c^0A, c^0B, c^0E$ ) were used to understand the surfaces topology.

The peculiarity of this phase diagram is a surface of liquid immiscibility, containing two maximum points (k,  $k^0$ ) on the edge  $kk^0$  and two minimum points (M, Q) at the same temperature of horizontal complex  $MQcqaq$ . As a solidus surface  $CcACqCEcb$  has a fold  $ck_0Cq$ , its contour resemble the contour of the liquidus  $Q_C$ , and the adjoining solvus surface  $cacqCEC^0Ec^0A$  has five points on counter - the same as the solvus surface  $acaqAeA^0Ea^0C$ .

The use of computer model permits to find the error in the visualization of phase diagrams with liquid immiscibility gap [21]. In [19] the solvus surfaces have

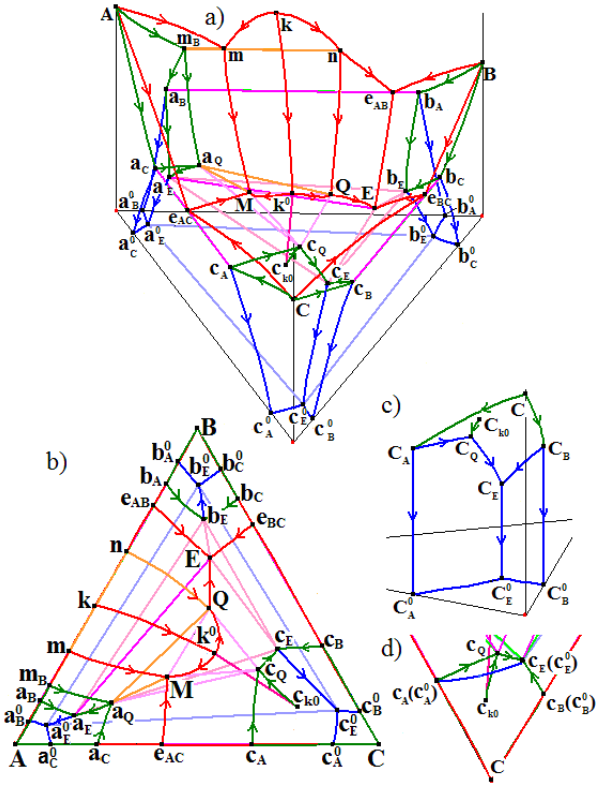


Fig. 2 – 3D model (a) and XY projection (b) of phase diagram with invariant monotectic equilibrium; fragment with orthogonal position of the  $L_1L_2$  lines on the contour of a solvus surface (c-d)

the orthogonal position of some lines on their contours (Fig. 2c-d – solvus lines  $CAC_0A$ ,  $CBC_0B$  and  $CEC_0E$  are projected into the points  $CA(C_0A)$ ,  $CB(C_0B)$ ,  $CE(C_0E)$ ). At that the projections of solvus surfaces with five points have not the closed contour: line  $CA(C_0A)$ -  $CE(C_0E)$  is absent (Fig. 2d).

**2.3 T-x-y diagram with monovariant syntectic equilibrium**

Given phase diagram has intermediate compound R in the AB binary system (Fig. 3a-b) and forms a surface of liquid immiscibility, 4 liquidus surfaces, 4 solidus surfaces, 10 solvus surfaces, 24 ruled surfaces ( $3i^r+10Q^r+5S^r+6V^r$ ) and 2 horizontal planes at temperatures of the ternary eutectics ( $E_1$  and  $E_2$ ).

In this case the initial data were the coordinates of the melting points of the initial components (A, B, C), the binary and ternary eutectics ( $e_{AB}$ ,  $e_{BA}$ ,  $e_{AC}$ ,  $e_{BC}$ ,  $E_1$ ,  $E_2$ ,  $e_r$ ), the points of the liquid-liquid immiscibility cupola (m, n, k,  $k^0$ ), the binary and ternary points of the surfaces of the solidus ( $a_B$ ,  $a_C$ , a;  $b_A$ ,  $b_C$ , b;  $c_A$ ,  $c_B$ ,  $c_{E1}$ ,  $c_{E2}$ ,  $c_r$ ;  $m_A$ ,  $n_B$ ,  $m_{E1}$ ,  $n_{E2}$ , r) and the solvus ( $a^0_B$ ,  $a^0_C$ ,  $a^0$ ;  $b^0_A$ ,  $b^0_C$ ,  $b^0$ ;  $c^0_A$ ,  $c^0_B$ ,  $c^0_{E1}$ ,  $c^0_{E2}$ ,  $c^0_r$ ;  $m^0_A$ ,  $n^0_B$ ,  $m^0_{E1}$ ,  $n^0_{E2}$ ,  $r^0$ ), and the R point of the binary compound. The solidus surface  $S_R(m_A R n_B n_{E2} r m_{E1})$  has the fold  $RR'$  (the point  $R'$  is located at a temperature corresponding to the lower point  $k^0$  of the liquid immiscibility surface) and point r of a local maximum (Fig. 3c-d). The surfaces of liquidus  $Q_R(e_{AB} m k^0 n e_{BA} E_2 e_r E_1)$  (Fig. 3c) and solidus  $S_c(C c_{ACE1} c_r c_{E2} C_B)$  (Fig. 3e) contain the points of maxima  $e_r$  and  $c_r$  too.

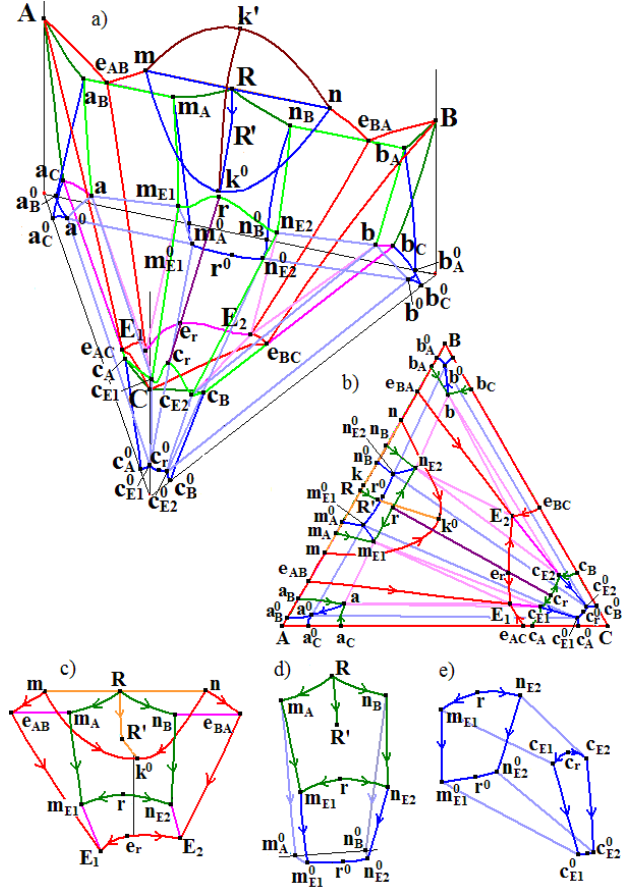


Fig. 3 – 3D model (a) and XY projection (b) of phase diagram with monovariant syntectic equilibrium and phase regions L+R (c), C+R (d) and R (e)

A transition of syntectic equilibrium  $L_1+L_2 \rightleftharpoons R$  into the monotectic one ( $L_1 \rightleftharpoons L_2+R$  or  $L_2 \rightleftharpoons L_1+R$ ) occurs in the phase region  $L_1+L_2+R$  [18, 22-23]. In this case two surfaces of 2-phase reactions  $L_1 \rightleftharpoons L_2$  and  $L_2 \rightleftharpoons L_1$  are appeared, and both these surfaces have a common generation segment  $k^0 R'$  (at the same time it is a tie-line between the  $L_1(L_2)$  and R phases (Fig. 4a). Phase region  $L_1+L_2+R$  is bounded by three ruled surfaces,  $i^r$  (with contour  $mk^0 n$ ),  $i^r_m$  ( $mRR'$ ),  $i^r_n$  ( $nRR'$ ), where the curves  $mk^0$  and  $nk^0$  are the crossing curves of immiscibility and liquidus surfaces.

The diagrams of material balance for arbitrary given mass center make possible to investigate the process of phase reaction type change in more detail. The composition G (0.46; 0.42; 0.12) (Fig. 4b) intersects the ruled surface  $i^r$  at  $T=620.5$  and moves from two-phase region  $L_1+L_2$  into three-phase region  $L_1+L_2+R$ . In phase region  $L_1+L_2+R$  the decrease of phases  $L_1$  and  $L_2$  portions, and the increase of phase R portion take place. At  $T=571.21$  the decrease of phase  $L_1$  portion stops and its increase begins, that is the change of syntectic equilibrium ( $L_1+L_2 \rightleftharpoons R$ ) to monotectic one ( $L_2 \rightleftharpoons L_1+R$ ) takes place. At  $T=558.49$  point G intersects the ruled surface  $i^r_m$  and phase  $L_2$  disappears. Below temperature 558.49 composition G falls into phase region L+R.

One, two or three surfaces of two-phase reaction can appear at the deformation of phase region  $L_1+L_2+R$  in such way that the line  $RR'$  is beyond the contour of line  $mk_0n$  in projection [23]. Usually this surface forms in the area of lines  $RR'$  and  $mk_0n$  intersection. More series deformation of the contour curves will produce two surfaces of two-phase reaction, adjoining to tie-line  $R^0k^0$ .

Table 2 – Surfaces of phase diagram (Fig. 3)

Name	Contour	Name	Contour
$i$	$mk_0nk^0$	$Q^{rAR}$	$eABABA E_1$
$Q_A$	$Ae_{AB}E_1e_{AC}$	$Q^{rRA}$	$eABMAME_1E_1$
$Q_B$	$Be_{BA}E_2e_{BC}$	$Q^{rAC}$	$eACACA E_1$
$Q_R$	$e_{AB}mk^0n_{eBA}E_2e_rE_1$	$Q^{rCA}$	$eACCACE_1E_1$
$Q_C$	$Ce_{AC}E_1e_rE_2e_{BC}$	$Q^{rBR}$	$eBABAB E_2$
$S_A$	$Aa_{ba}aC$	$Q^{rRB}$	$eBANBNE_2E_2$
$S_B$	$Bb_{abb}c$	$Q^{rBC}$	$eBCbcb E_2$
$S_R$	$mARNbNE_2rME_1$	$Q^{rCB}$	$eBCCBCE_2E_2$
$S_C$	$CCACE_1C_rCE_2CB$	$Q^{rCR}$	$E_1CE_1CE_2E_2$
$V_{AB}$	$abaa^0a^0B$	$Q^{rRC}$	$E_1ME_1NE_2E_2$
$V_{AC}$	$acaa^0a^0C$	$S^{rAR}$	$abMAME_1a$
$V_{BA}$	$babb^0b^0A$	$S^{rBR}$	$baNBNE_2b$
$V_{BC}$	$bccb^0b^0C$	$S^{rAC}$	$accACE_1a$
$V_{CA}$	$cACE_1C^0E_1C^0A$	$S^{rBC}$	$bCCBCE_2b$
$V_{CB}$	$CBCE_2C^0E_2C^0B$	$S^{rRC}$	$ce_1ME_1NE_2CE_2$
$V_{CR}$	$CE_1C_rCE_2C^0E_2C^0_rC^0E_1$	$V^{rAR}$	$ame_1m^0E_1a^0$
$V_{RA}$	$mAME_1m^0E_1m^0A$	$V^{rBR}$	$bnE_2n^0E_2b^0$
$V_{RB}$	$nBNE_2n^0E_2n^0B$	$V^{rAC}$	$ace_1C^0E_1a^0$
$V_{RC}$	$mE_1rNE_2n^0E_2r^0m^0E_1$	$V^{rBC}$	$bCE_2C^0E_2b^0$
$i^r$	$mnk^0$	$V^{rRCm}$	$ce_1ME_1m^0E_1C^0E_1$
$i^r_m$	$mR(R,r)^0$	$V^{rRCn}$	$ce_2NE_2n^0E_2C^0E_2$
$i^r_n$	$nR(R,r)^0$	$H_{E_1}$	$ace_1ME_1$
		$H_{E_2}$	$bce_2NE_2$

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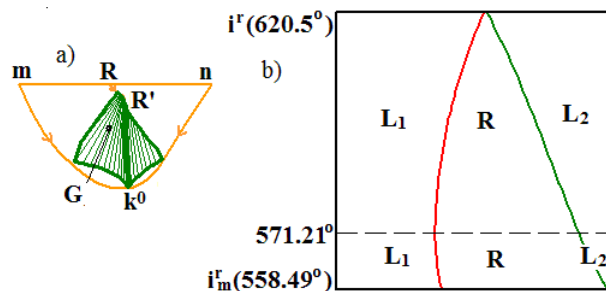


Fig. 4 – Surfaces of two-phase reaction for phase region  $L_1+L_2+R$  (a); the material balances for the compositions G (b)

## 5. CONCLUSIONS

Using the mathematical methods of phase diagrams elements descriptions at the computer software allowed to simulate the whole geometrical structure of diagram on the basis of minimal initial information. At that the effectiveness of corresponding physics-chemical systems study is increased: the visualization of diagram geometrical model and its sections has been possible, the founding of information about the results of initial components interaction for the work content reduction of new materials creation has become available. Such software makes possible to solve the problem of heterogeneous materials computer-aided design.