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Modeling and control of complex systems over finite fields

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Dynamical systems over finite fields [1] provide a natural mathematical framework for modeling and computer simulation of complex systems [2, 3]. The dynamics of finite dynamical systems is described by the local interactions of a large number of heterogeneous entities, which can be in one of finitely many different states and interact with each other and the environment by means of deterministic or stochastic rules. The global dynamics emerges from the local dynamics and interactions among the local entities described by the finite digraph; such a scheme can be found, for example, in social, socio-technical, and biological systems.

The general description of the approach that we present is as follows. We consider a finite field F_q, where q stands for the number of elements in the field and is a power of prime characteristic of the field [4].

Let $X = (x_1, x_2,\ldots, x_s)$ be a collection of variables representing entities in the system and taking on values from the field F_q. The elements u_i of a finite nonempty space $U = (u_1, u_2,\ldots, u_s)$ of all bounded maps $u: R \to U \subseteq F_q^s$ define control functions associated with x_i.

The phase space of the system is described by the finite digraph $G = (V, E)$, where $|V| = s$ and each node is associated with the variable; the edges describe the connections between variables – there is the directed edge from x_i to x_j if x_j depends on x_i. The local dynamics of x_i is a transition function $\phi_i : F_q^{r+1} \times U \to F_q$, where r is the in-degree of the vertex i.

The global dynamics $\Phi : F_q^s \times U \to F_q^s$ assembled by local functions defines the finite dynamical system with control. The overall dynamics is generated by the iterations of Φ, which represents the transitions between system configurations $\Gamma = (\gamma_1, \gamma_2,\ldots, \gamma_s)$, $\gamma_i \in F_q^s$; or, alternatively, it can be defined as a mapping $\Gamma : X \to F_q^s$.

The global transition function is constructed using the operations on the set of local functions. We define several operations: parallel composition of local functions, sequential composition of functions, as well as the “mixed” operation which can be justified by the idea of dividing the variables into groups with one of the operations within a group and defining the other operation on the set of the groups.

The finite dynamical systems with control capture several important characteristics of the models: all state variables in the model change their state discretely; all state variables take on only finitely many different states; all state variables are subject to controls, which take on finitely many different values; controls are either internal, from within a system, or external, from a controller (user, observer, supervisor).

Moreover, finite dynamical models naturally capture the essential elements of computer simulation [5]. The mathematical mappings, used to represent the dynamics of each entity in the system, can be described by automata. The local dependence of each entity on the states of the entities in its neighborhood reflects the intuitively comprehensible fact that individual objects in a real system usually depends on its local environment and can be in some sense controlled by neighboring objects. This is quite natural for biological and social systems, in which the dependency graph is the graph of mutual influences, and the neighborhood of an agent can model the influence of a group of agents on some other concrete agent.

The simulation process can solve several questions as to controllability (the ability to reach any one state from any other), reachability (the ability to reach the set of states to which the system can be steered), accessibility (the ability to reach a subset of the state space from any given initial state).

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