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NONLINEAR EFFECTS IN SEMI-CLASSICAL MODEL OF SOLID-STATE LASER

G.P. Kovalenko, S.V. Kolomiets
Sumy National Agrarian University,
160 Kirova Str., Sumy , 40021 Ukraine, e- mail: s_kolomiets@mail.ru

This paper is devoted to the theoretical research of ways of initiation of unsteady condition in semi-classical model of single-mode of solid-state laser. The bifurcation analysis of semi-classical model of solid – state laser with inertialess filter and quadratically-nonlinear element is executed. The stability criterion of periodic oscillations, periodic solution in quadratic approximation and analytical expression of a limit cycle in a linear approximation are constructed. The numerical calculations of members of a limit cycle are adduced.

Semi-classical model of single-mode laser with filter is analyzed [1]:

$$\begin{cases} \dot{x} = Gx\left(y - 1 - b(1+\rho x)^{-1} - rx\right) \\ \dot{y} = A - y(x+1) \end{cases}, \qquad (1)$$

where x - is photon field density, y - is inversion, ρ - is ration of active medium saturation to filter saturation, r - is reduced coefficient of nonlinear interaction of resonator with nonlinear element, G - is high parameter in the theory of class B lasers, A - is pumping parameter, $b = \dfrac{Q_0 l_2}{k_n l_1}$, l_1, l_2 - distance from filter to resonator ends, Q_0 - is absorption constant of filter, $k_n = \eta - \ln(r_1 r_2 / 2l)$, l - is active medium length , η is defined passive loss, r_1, r_2 - are mirrors reflectivity. The parameters, phase coordinates and time are dimensionless.

The objectives of the work are to determine the conditions of initiation of Hopf's bifurcation in dynamic system, to study stability of periodical oscillations, which arise as a result of the loss of stationary solution stability, when one of the model's parameters passes through its own bifurcation value, and to formulate a periodical solution of the dynamic system. For asymptotic integration the system of differential equations algorithm of bifurcation of cycle initiation [2] is used. This method also allows studying the problem of the limiting cycle stability.

To study the problem of stability of periodical oscillations according to the bifurcation algorithm of cycle initiation, Φ value is build, the real part of which determines the Floquet's ratio sign. Presence of the high parameter G in semi-classical models of solid-state lasers allows singling out only three items from the cumbersome expression to determine the real part $\operatorname{Re}\Phi$ that leads to obtaining the criterion of stability:

$$\operatorname{Re}\Phi = \frac{G}{8Ax_c}\left(2A_1^2\alpha^2 + A_1 A - 3AD_1 x_c\right) < 0,$$

$$\alpha = x_c + 1, A_1 = r - \frac{b\rho}{(\rho x_c + 1)^3}, D_1 = \frac{\rho^2 b}{(\rho x_c + 1)^4}. \qquad (2)$$

In case of bifurcation parameters b and r it is possible to find an interval of stability for parameter ρx_c.

The periodical solution of dynamic system (1) construct to the special formula [2]. The analysis of the found periodical solution of the system (1) is made in linear approximation by the low functional parameter ε that allows representation of the found solution in the following form:

$$x - x_c = \varepsilon \cos 2\theta,$$

$$y - y_c = -\frac{\varepsilon}{Gx_c}(\alpha \cos 2\theta + \omega_0 \sin 2\theta) = -\frac{\varepsilon N}{Gx_c}\cos(2\theta - \varphi), \quad (3)$$

$$\omega_0 = \sqrt{\frac{GAx_c}{\alpha}}, \quad \varphi = arctg\frac{\omega_0}{\alpha}, \quad N = \sqrt{\alpha^2 + \omega_0^2}$$

The extraction from relation's (3) parameters θ has allowed finding a curve, which one describes a limit cycle as a first approximation.

It should be noted that, points of bifurcation play an important role when analyzing properties of the dynamic system. In particular, bifurcation of limit cycle initiation assigns the conditions for development of auto-oscillations. The algorithm of cycle initiation bifurcation we applied for the analysis gives opportunity to solve problems concerning the conditions of bifurcation occurrence of cycle initiation, to find the stability criterion of periodic oscillations, period, modulation amplitude and phase and also to construct the approximate solution of dynamic system. Therefore, the used method has wide prospects for further application to the analysis of other laser models of similar type.

References

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[2] V. Hassard, N. Kazarinov, I.Van. Theory and Applications of Bifurcation of Cycle Initiation, Mir, Moscow, (1985) (in Russian).