МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ СУМСЬКИЙ ДЕРЖАВНИЙ УНІВЕРСИТЕТ

### ІНФОРМАТИКА, МАТЕМАТИКА, АВТОМАТИКА

## IMA :: 2017

#### МАТЕРІАЛИ та програма

#### НАУКОВО-ТЕХНІЧНОЇ КОНФЕРЕНЦІЇ

(Суми, 17-21 квітня 2017 року)



Суми Сумський державний університет 2017

# Mathematical modelling of disk piezoelectric transformer with ring electrode in primary electrical circuit

Bazilo C.V., Ph. D., *Associate Professor* Cherkasy State Technological University, Cherkasy

Currently, there are no reliable and valid methods of constructing of mathematical models of piezoelectric transformers, which could be used as a theoretical basis for characteristics and parameters calculating of this class of functional elements of modern piezoelectronics.

In most papers the described methods of piezoelectric transformers models constructing are mostly based on the use of equivalent electrical circuits and it does not allow analyzing of stress-strain state of solids with the piezoelectric effects.

In this work, using the simplest example of axially symmetric radial oscillations of the piezoelectric disk the principles of mathematical models constructing that are sufficiently adequate to real devices and occurring physical processes are set out.

Disk piezoelectric transformer (Fig. 1), primary electrical circuit of which consists of electric potential difference generator  $U_1 e^{i\omega t}$  (where  $U_1$  is an amplitude value of electric potential difference;  $i = \sqrt{-1}$  is an imaginary unit;  $\omega$  is an angular frequency; t is a time) with output electrical impedance  $Z_g$  and ring electrode (position 1 in Fig. 1).



Figure 1 - Calculation scheme of disk piezoelectric transformer

The secondary electrical circuit consists of an electrode in the form of a circle (position 2) with connected electronic circuit to it with input electrical impedance  $Z_n$ , on which an electric potential difference  $U_2e^{i\omega t}$  is formed.

The primary and secondary circuits of piezoelectric transformer do not have a galvanic connection. The energy exchange between the primary and secondary circuits is carried out by means of axisymmetric radial vibrations of the piezoceramics material particles in the volume of thickness polarized disk (position 3 in Fig. 1).

Mathematical model of piezoelectric transformer with ring electrode in the primary circuit can be written as

$$K(\omega,\Pi) = \frac{U_2}{U_1} = \frac{K_2(\Omega,\Pi)}{1 - i\omega C_0^{\sigma} Z_g K_3(\Omega,\Pi)},$$
(1)

where

$$\begin{split} K_{2}(\Omega,\Pi) &= \frac{2f_{e}(\omega)K_{31}^{2}A_{12}\left[J_{1}(\Omega R_{1}/R)/(\Omega R_{1}/R)\right]}{1-2f_{e}(\omega)K_{31}^{2}A_{11}\left[J_{1}(\Omega R_{1}/R)/(\Omega R_{1}/R)\right]};\\ K_{3}(\Omega,\Pi) &= \\ &= \frac{2K_{31}^{2}}{1-\beta^{2}}\left\{\left[K_{2}(\Omega,\Pi)A_{41}+A_{42}\right]J(\Omega)+\left[K_{2}(\Omega,\Pi)A_{51}+A_{52}\right]N(\Omega)\right\}-1\right\}\\ J(\Omega) &= \left[J_{1}(\Omega R_{3}/R)-\beta J_{1}(\beta \Omega R_{3}/R)\right]/(\Omega R_{3}/R);\\ N(\Omega) &= \left[N_{1}(\Omega R_{3}/R)-\beta N_{1}(\beta \Omega R_{3}/R)\right]/(\Omega R_{3}/R); \end{split}$$

 $\Pi$  is a set of electrical, geometrical, physical and mechanical parameters of the transformer;  $K_{31}^2$  is a squared electromechanical coupling coefficient for the mode of radial oscillations of thickness polarized piezoceramic disk material particles; constants  $A_{ij}$  define the radial displacements of disk material particles under the electrodes;  $f_e(\omega)$  is a switching on function;  $\beta$  is a geometrical parameter of the ring;  $C_0^{\sigma}$  is a static electrical capacitance.

Expression (1), which determines the transfer ratio of piezoelectric device, has a structure which is typical for electronic devices with negative feedback. It is clearly seen that the depth of feedback is directly proportional to the value of the signal source output impedance  $Z_g$ . If the value of Zg = 0 the feedback disappears and transfer ratio is completely determined by a frequency dependent function K2( $\Omega,\Pi$ ).