

The Liebman Process for Distribution of the Information Flows of the Engine Automatic Control Systems

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The paper describes the Liebman process of creating distributed and reliable ACS aircraft power plant within the identifying information flows in the interaction of nodes, which allows, on the one hand, to organize optimal control with adaptation to changing external conditions and, on the other hand, to take timely and correct decisions when violations occur in the power plant or in the control part of the ACS. In the case of the distributed information flows of the electronic control system of aviation engine, different equation techniques have been considered and, depending on the given part of the system, their useful types have been defined: elliptical, hyperbolic, parabolic. According to the Liebman averaging process the practical iteration with the sufficient number of special computing templates has been proposed for estimating the accuracy of the grid-derived solution of the distributed information flow of the engine automatic control system. The scheme of the information flow processing of the electronic system based on the developed algorithm and practical results in the form of programming language and graph visualization has been presented.

Keywords: Liebman process, Aviation engine, Electronic control system, Information flows, Distributed system, Elliptical equations, Computing templates, Visualization.

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1. INTRODUCTION

The development of computer technology has led to the appearance of technology for the design and prediction of complex processes and objects, providing researchers with an effective tool for mathematical modeling of electronic digital Automatic Control System of Gas Turbine Engine (ACS GTE).

Modern mathematical models are not only limited to the problem of finding regularities, but also require powerful methods of realization such as the development of efficient algorithms involving numerical methods. The software package of mathematical modeling of the digital ACS [1-4], built on a modular principle, includes models of information-measuring channels, actuators, computing part of Onboard Digital Computing Machine (OnBDCM), object of control, interferences and failures in the information-computing process and other accessorial modules. Models of information-measuring channels take into account the dynamic characteristics of the sensors, the quantization of the measurement signals by level, the discreteness over time and the effects associated with the process of analog-to-digital conversion. The information channel model is also a model of interference. The background component of random interference in channels, caused by measurement errors, transformations, is simulated by discrete Gaussian noise in the input circuits of the digital part of the OnBDCM and the shaper for each channel [4-6].

Models of actuators (mechanisms) take into account their dynamic characteristics and performance limitations. The mathematical model of the computational part of the OnBDCM is implemented in the form of differential equations, which allow to take into consideration the clock cycle in OnBDCM and to simulate the memory cells. The model of failure in the information-processing process provides the possibility to change the coefficients of the differential equations and the

content of memory cells.

Interface for tuning the parameters of the output converters that form the control signals for actuators in the fuel flow control channels, the fan position and the compressor position, is implemented as a table in a separate graphic window. The parameter setting interface in this model is implemented as a series of graphical windows: compressor speed controller, fan speed limiter, gas temperature behind the turbine, compressor pressure, etc. The considered model of digital ACS can work together with mathematical models of GTE of different level of complexity.

The software package allows to select the parameters of digital filters, algorithms for recovering information by indirect parameters, algorithms for estimation by probability criteria and other algorithms for system protection, to investigate the work in a closed loop of the digital control system at different levels and time of start and end of interference in information-measuring channels.

The research of such electronic control systems of aviation engine and their development causes the development of special methods (processes of averaging) and means of mathematical modeling of their work taking into account the influence on them of electromagnetic and other interferences, instability of onboard power supply, occurrence of failures, etc. The use of such mathematical models in the stages of preliminary development and selection of system parameters during the period of commissioning and operation allows to improve the quality and reduce the time and resources for development. Great contribution to the development of electronic control system of the aviation engines has been made by scientists: Kreiner A., Ramsey J.W., Culley D., Letzau K., Holberg F.D., Gyrevich O.S., Petuhov A.A. [1-7] and SNECMA, Rolls-Royce, MTU Enterprises within the project OBIDICOTE (On Board Identification Diagnosis and Control of gas Turbine Engine) [1-4].

The aim of this research is to define the special

methods and processes of mathematical modeling of the electronic control systems of aviation engine taking into account the influence of electromagnetic and other interferences, instability of onboard power supply, occurrence of failures.

2. LIEBMAN METHOD FOR INFORMATION FLOW IN THE ELECTRONIC CONTROL SYSTEM OF AVIATION ENGINE

2.1 Differential Equation Technique

In the general case for the information-distributed flows of the aviation engine, the partial differential derivative equation is [7, 8]:

$$F(x, y, u, u_x, u_y, u_{xx}, u_{xy}, u_{yy}) = 0, \tag{1}$$

where x, y are independent variables, u is the required function, $u_x, u_y, u_{xx}, u_{xy}, u_{yy}$, are its first and second partial derivatives on the arguments x and y .

The solution of equation (1) is a function $u = u(x, y)$ that converts this equation into an identity. The intersection graph is the surface in space $Oxyu$ (integral surface) (Fig. 1).

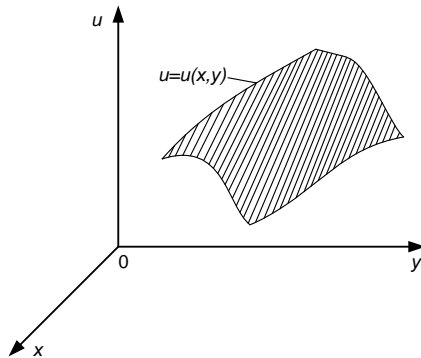


Fig. 1 – Integral surface of the equation (1)

Eq. (1) is called linear, more precisely, completely linear, if it is of the first degree with respect to the desired function and all its derivatives do not contain their products, that is, if this equation can be written in the form [8]:

$$A \frac{\partial^2 u}{\partial x^2} + 2B \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2} + a \frac{\partial u}{\partial x} + b \frac{\partial u}{\partial y} + cu = F(x, y), \tag{2}$$

Moreover, the coefficients A, B, C, a, b, c can only depend on x and y . In the particular case, if these coefficients are independent of x and y , then Eq. (2) will be a linear differential equation with constant coefficients. Let us consider in more detail the linear differential Eq. (2).

Let be $D = AC - B^2$ is the discriminant of the equation. Depending on the sign of the function D , linear differential Eq. (2) belongs in the given area to one of the following types:

- $D > 0$ – elliptical type;
- $D = 0$ – parabolic type;
- $D < 0$ – hyperbolic type;
- D does not store a permanent sign, has mixed type.

The type of linear Eq. (2) is its important feature and is preserved in any non-degenerate transformation

$$\xi = \phi(x, y), \quad \eta = \psi(x, y), \tag{3}$$

that is, the Jacobian

$$\frac{\partial(\phi, \psi)}{\partial(x, y)} \neq 0.$$

With linear differential equation

$$A \frac{\partial^2 u}{\partial x^2} + 2B \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2} + a \frac{\partial u}{\partial x} + cu = F(x, y) \tag{4}$$

the ordinary differential equation is bound

$$A(dy)^2 - 2Bdxdy + C(dx)^2 = 0, \tag{5}$$

which is called characteristic; the solutions of Eq. (5) are called the characteristics of Eq. (4).

For Eq. (4) of hyperbolic type, there are two families of characteristics (Fig. 2): $\phi(x, y) = C_1$ and $\psi(x, y) = C_2$.

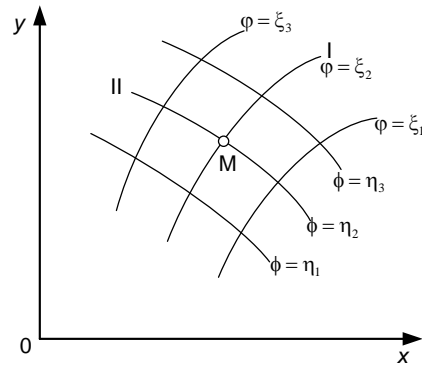


Fig. 2 – Families of characteristics of the hyperbolic equation

Conducting in Eq. (4) the transformation $\xi = \phi(x, y), \eta = \psi(x, y)$, that is, assuming the parameters of these families as new curvilinear coordinates, the canonical form of the hyperbolic type equation can be obtain:

$$u_{\xi\eta} + \alpha(\xi, \eta)u_\xi + \beta(\xi, \eta)u_\eta + \gamma(\xi, \eta)u = f(\xi, \eta).$$

Eq. (4) of the parabolic type has one family of characteristics $\phi(x, y) = C$.

As a result of the transformation $\xi = \phi(x, y), \eta = y$, the parabolic type equation is reduced to the canonical form:

$$u_{\eta\eta} + \alpha(\xi, \eta)u_\xi + \beta(\xi, \eta)u_\eta + \gamma(\xi, \eta)u = f(\xi, \eta).$$

Finally, Eq. (4) of the elliptic type admits two families of complex characteristics:

$$\begin{aligned} \phi(x, y) + i\psi(x, y) &= C_1, \\ \phi(x, y) - i\psi(x, y) &= C_2. \end{aligned}$$

Conducting the transformation $\xi = \phi(x, y), \eta = \psi(x, y)$, the canonical form of the equation of elliptic type can be obtained:

$$\Delta u + \alpha(\xi, \eta)u_\xi + \beta(\xi, \eta)u_\eta + \gamma(\xi, \eta)u = f(\xi, \eta),$$

$\Delta u = u_{\xi\xi} + u_{\eta\eta}$ is the Laplace operator.

The simplest equation of the elliptic type $\Delta u = 0$ is called the Laplace equation [7, 8]. The nonuniform Laplace equation $\Delta u = f(\xi, \eta)$ is called the Poisson equation. The partial differential derivative has, in the general case, innumerable solutions. Therefore, if a physical process is described using the partial derivative equation, then some additional conditions must be attached to the equation to uniquely characterize the process. In the simplest case, this additional data consists of initial and boundary conditions. These conditions can be distinguished only if one of the independent variables of the differential equation plays the role of time and the other – of the spatial coordinate (for the case of two independent variables). In this case, the conditions relating to the initial moment of time are called initial, and the conditions relating to the fixed values of coordinates are boundary.

2.2 Liebman Process

According to the Liebman averaging process and analysis the information flows of the engine automatic control system [9, 10], choosing the initial approximations $u_{ij}^{(0)}$, successive approximations $u_{ij}^{(k)}$ for the inner nodes (x_i, y_i) of the grid S_h are determined by the formula [7, 8]:

$$u_{ij}^{(k)} = \frac{1}{4} [u_{i-1,j}^{(k-1)} + u_{i+1,j}^{(k-1)} + u_{i,j-1}^{(k-1)} + u_{i,j+1}^{(k-1)}] \quad (k=1,2,\dots). \quad (14)$$

As for the boundary nodes A_h of the grid S_h , the values of the function $u(A_h)$ in these nodes are consistently corrected by the formulas of linear interpolation:

$$\begin{aligned} u^{(0)}(A_h) &= u(A) = \phi(A), \\ u^{(k)}(A_h) &= u(A) + \frac{u^{(k-1)}(B) - u(A)}{h + \delta} \delta \quad (15) \\ (k &= 1, 2, \dots), \end{aligned}$$

where A is the closest to A_h the boundary point, $\Gamma(u(A) = \phi(A))$, B is the closest to A_h the inner node of the grid S_h (Fig. 3) and δ is the removal of the node A_h from the point A . Moreover, $\delta > 0$ if A_h is the inner point of the area G , and $\delta < 0$ if A_h is the outer point of the area G . In the particular case, if the node A_h lies on the boundary $\Gamma (A_h \equiv A, \delta = 0)$, it can be found exactly

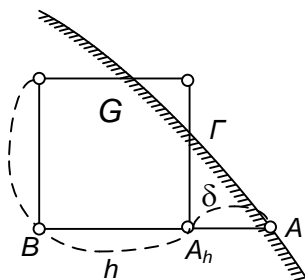


Fig. 3 – Grid node for averaging process

$$u^{(k)}(A_h) = u(A) = \phi(A).$$

In practice, after a certain step k , $u^{(k)}(A_h)$ can be considered unchanged (for example, if these values are set with a given accuracy).

By default values $u_{ij}^{(0)}$, any number system can be taken theoretically. However, it should be borne in mind that due to the maximum principle, inequalities must be satisfied for the values of the desired function $u(x, y)$

$$m \leq u_{ij} \leq M,$$

where the boundaries of Γ : $m = \min \phi(P)$, $M = \max \phi(P)$. Therefore, it is reasonable to assume $m \leq u_{ij}^{(0)} \leq M$.

Practically, the choices $u_{ij}^{(0)}$ are roughly solved for the Dirichlet problem in the area G by means of a large grid, and then the values found are used to solve the Dirichlet problem on a given small grid. Linear interpolation is usually used to begin the process.

It turns out that for any step h of the grid, the Liebman process regardless of the choice of initial values converges, that is, there is $\lim_{k \rightarrow \infty} u_{ij}^{(k)} = u_{ij}$, moreover, the error of the approximate solution has order $O(h^2)$. For practical iteration, it is useful to prepare a sufficient number of special computing templates (Fig. 4) [6-8].

The calculation scheme must be stable, that is, rounding errors should not increase indefinitely.

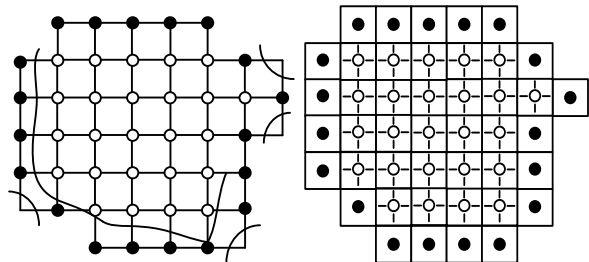


Fig. 4 – Computing templates for the distributed information flows [8]

3. ENGINE DYNAMICS INFORMATION FLOW SYSTEM

3.1 Grid Method

After reviewing, the scheme of the information flow process of the aviation engine electronic system based on the developed algorithm and practical results can be considered.

Algorithm. STEP 1. In the main program block in dialog mode, enter the following variables: a, b are the size of the area; e is the error of solution of the system of difference equations; m, n are the numbers of partitions of the area Γ along the coordinates x and y . In the *for* loop, define boundary conditions at each node of the selected grid.

STEP 2. Assign values T_{ij} in the internal nodes to a numerical value equal to the average of all values of the boundary conditions.

STEP 3. Recalculate the value at all internal points of the grid by replacing the old value with the average of

four adjacent points before the condition $|T_{i,j}^{k+1} - T_{i,j}^k| < \varepsilon$ is fulfilled.

STEP 4. Output the value of the solution at all internal nodes, write the result to a file, and then use the program "Surfer8" to get a graphical solution [11].

It is not very important how the account process will be organized, but it is usually carried out in rows (or columns). After a few operations, the process will come to an approximate solution to the problem.

Practical results. Considering the squares $ABCD$ with vertices $A(0;0)$, $B(0;1)$, $C(1;1)$, $D(1;0)$, the steps of the grid along the axes OX and OY can be considered equal to $h = k = 0.2$. Boundary conditions are:

$$T|_{AB} = 45y(1-y); T|_{BC} = 25x; T|_{CD} = 25;$$

$$T|_{AD} = 25x \sin(\pi x / 2);$$

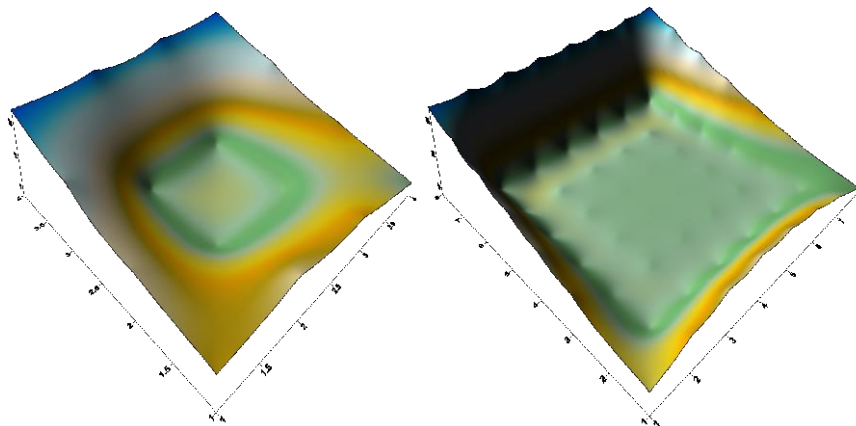


Fig. 5 – Engine information flow process with the number of iterations of 4 (left side) and of 36 (right side)

4. CONCLUSIONS

In recent years, in the development of aircraft engine industry, there has been a tendency to further complicate the concepts of power plants (PP) designed for aircraft of various classes. This leads to a significant complication of the information flows of electronic control systems of aircraft engines with ever-increasing requirements for the quality and reliability of the functioning of automatic control systems (ACS) in a wide range of changes in flight conditions, operating modes and external disturbances.

A number of methods for solving the problems of terminal control of PP under conditions of parametric uncertainty of the interval type and the presence of limited interference in the channels for measuring phase coordinates are of an original character. Particularly noteworthy are the learning algorithms that represent various modifications of the pattern recognition training procedures in which the information obtained as a result of testing a

The program to solve this problem is written in the programming language in the training compiler:

```
begin
  for i: = 1 to m do begin x: = a + (b - a) × (i/m);
  t[1,i]: = 25 × x; end;
  for j: = 1 to n do begin y: = a + (b - a) × (j/n);
  t[j,1]: = 45 × y (1 - y); end;
  for i: = 1 to m do begin x: = a + (b - a) × (i/m);
  t[n,i]: = 25 × x × sin(π × x / 2); end;
  for j: = 1 to n do begin y: = a + (b - a) × (j/n);
  t[j,m]: = 25; end;
  begin
    r: = (t[i - 1,j] + t[i + 1,j] + t[i,j - 1] + t[i,j + 1]) / 4;
    if abs(r - t[i,j]) < e × abs(r) then k: = 1; tt[i,j]: = r;
  end.
```

controlled process plays the role of an external “teacher”.

On the other hand, methods for solving equations with spatial and partial derivatives are determined that contain an unknown function that must be defined and used as training for pattern recognition during mathematical modeling of electronic control systems.

Progress in the field of high-performance computers has led to the creation of new numerical methods with their presentation at the “epsilon-delta” level and the combination of several different approaches to modeling electronic engine control systems such as elliptic, hyperbolic and parabolic equations for solving equations by the finite difference method.

For design and investigation of the information flows of the electronic engine control system, the Liebman method has been presented. The experimental results for the Liebman process with template of type “cross” and iteration procedures in the form of programming language and graph solution “Surfer” have been presented.

REFERENCES

1. O.S. Gurevich, *Sistemy avtomaticheskogo upravleniya aviatsionnyimi gazoturbinnymi dvigatelyami*, 264 (Moscow: Toru Press: 2010) (in Russian).
2. Securaplane Technology Inc. *Wireless technology intra-aircraft wireless data bus for essential and critical applications*. Access mode: <https://www.securaplane.com/>
3. *EUROCAE – standarts for future aviation*. Access mode: <https://www.eurocae.net/>
4. A.S. Tanenbaum, *Distributed Systems: principles and paradigms*, 702 (CreateSpace Independent Publishing Platform: 2016).

5. Yu.A. Klymentovskiy, *Systemy avtomaticheskogo upravleniya sylovymy ustanovkamy letatelnykh apparatov* (Kyiv: 2001) (in Russian).
6. C.E. Fisher, *Gas Path Debris Monitoring – A 21-st Century PHM Tool* (Stewart Hughes Limited).
7. S.J. Farlow, *Partial Differential Equations for Scientists and Engineers*, 448 (Dover Publications: 1993).
8. I.V. Miroshkina, O.A. Palagina, *Chislenni metodi: posibnik*, 116 (Cherkasy: ChSTU: 2017) (in Ukrainian).
9. S.S. Tovkach, *Electron. Control Syst.* 3, 29 (2019).
10. S.S. Tovkach, *J. Nano- Electron. Phys.* 11 No1, 01016 (2019).

Процес Лібмана для розподілу інформаційних потоків систем автоматичного керування двигуном

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У статті описаний процес Лібмана зі створення розподілених і надійних САК СУЛА в рамках ідентифікації інформаційних потоків при взаємодії вузлів, що дозволяє, з одного боку, організувати оптимальне управління з адаптацією до зовнішніх умов, а з іншого боку, вживати своєчасні і правильні рішення при виникненні відхилень на силовій установці або в керуючій частині САК. Для розподілених інформаційних потоків електронної системи управління авіаційним двигуном були розглянуті різні рівняння і, в залежності від заданої частини системи, визначені корисні їх типи: еліптичні, гіперболічні, параболічні. Відповідно до процесу усереднення Лібмана запропонована практична ітерація з достатньою кількістю спеціальних обчислювальних шаблонів для оцінки точності рішення, отриманого з сітки, розподіленого інформаційного потоку системи автоматичного керування двигуном. Представлена схема обробки інформаційних потоків електронної системи на основі розробленого алгоритму і практичної реалізації у вигляді мови програмування і графічної візуалізації.

Ключові слова: Процес Лібмана, Авіаційний двигун, Електронна система керування, Інформаційні потоки, Розподілена система, Еліптичні рівняння, Обчислювальні шаблони, Візуалізація.