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# Mathematical study of energy characteristics of centrifugal pump with single-vane impeller

Trofimenko P. E., Naida M. V., Khomenko A. V.

*Sumy State University,*
*2 Rimsky Korsakov Str., 40007, Sumy, Ukraine*

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In the article, the design of a centrifugal pump with a single-vane impeller is described and a theoretical calculation of such a pump is provided. The analysis of experimental investigations and the comparison with theoretical calculations are carried out. For the first time, the operating characteristics of a pump with a single-vane impeller of this type are obtained for different values of the rotation frequency.

**Keywords:** *impeller, pump, fluid characteristics, pressure, flow, power.*

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## 1. Introduction

The need for reliable and simple pumping equipment for the processing of both pure liquids and hydraulic mixtures exists in a number of industries: oil production and processing, chemical, aviation, medical and food, biotechnology, energy and, no less important, in military-industrial complex. To date, the problem of creating pumping equipment capable of working efficiently in a wide range of flows, pressures, taking into account the characteristics of the pumped medium, is quite acute. In recent years, the latest technologies have been actively introduced in the field of pump engineering aimed at solving urgent problems in this area. The properties and composition of the pumped fluids significantly change both the characteristics of the networks and the requirements for the characteristics that pump equipment uses [1–13].

If we use the speed coefficient generally accepted for vane pumps ($n_s$) of their flow part, then this is the range of parameters with $35 \leqslant n_s \leqslant 1200$. The value of the specific speed coefficient of the flow part of dynamic pumps is determined by the dependence [1–6]:

$$n_s = \frac{3.65 n \sqrt{Q}}{H^{3/4}}, \tag{1}$$

where $n$ is the rotor rotation frequency, rpm; $H$ is the pump head, m; $Q$ is the pump flow rate, $\mathrm{m}^3/\mathrm{s}$.

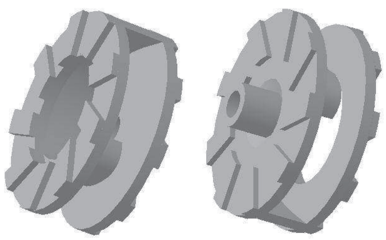

**Fig. 1.** Single-vane (left) and two-bladed (right) impellers of a centrifugal pump.

During the operation of centrifugal pumps of traditional design on multi-phase hydraulic mixtures as a result of overlapping the flow cross-sections of the flow parts of such pumps with solid particles or gas bubbles, a breakdown occurs, which, in turn, leads to significant economic losses for organizations operating these pumps. Transfer of multi-phase media requires the use of special vane pumps that are insensitive to the composition of the pumped fluid. In the specific speed range $130 \leqslant n_s \leqslant 300$ for pumping solid-liquid and gas-liquid mixtures, centrifugal pumps with single- and two-bladed impellers are widely used (Fig. 1).

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Fig. 2 shows the ranges of use of centrifugal pumps with single-, two- and seven-bladed impellers on gas-liquid mixtures. From the characteristics, it is seen that centrifugal pumps with single-vane impellers operate quite efficiently on gas-liquid mixtures with a high gas content (up to 50% of the total volume of the hydraulic mixture). Centrifugal pumps with two blades work on gas content range from 15 to 34% and with seven blades work on gas content range from 0 to 15%, after which pump parameters are disrupted.

During the design of the pump, and especially when calculating the impeller, the number of impeller blades is determined, as well as the influence of the final number of blades on the pump head. A small number of blades are associated with a small friction area and simplifies the production of blades [14, 15]. At the same time, the pressure on the blade increases and, naturally, the difference in speeds on both sides of the blade increases. Which in turn leads to a double conversion of speed, which is inevitably associated with pressure on the blades. As a result of this, separation zones and loss of separation increase. It should be added that the increasing pressure on the blades also reduces the suction capacity of the pump, that is, the risk of cavitation increases [1–6].

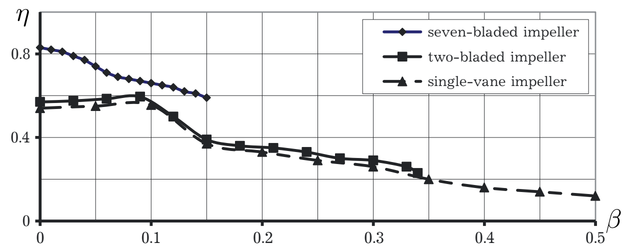


**Fig. 2.** Comparative characteristics of centrifugal pumps that run on gas-liquid mixtures.

## 2. Formulation of problem

Based on the above, it was decided on the need for further study of a centrifugal pump with a single-vane impeller. Despite the low efficiency, single-vane impellers of centrifugal pumps are used in industry, therefore, their research continues [16–20]. In the experiments, the design of the centrifugal-vortex stage are used (Fig. 3: 1, impeller; 2, front vortex stage; 3, rear vortex stage; 4, impeller housing). It has the same properties as traditional centrifugal vortex pumps, but is largely devoid of their main drawbacks (poor operational reliability and significant axial forces) [7].

The specified stage in its design refers to the small-sized type of working bodies of dynamic pumps. This stage has a single-blade centrifugal impeller (Fig. 4), equipped with additional vortex channels, which serve as a closed vortex impeller, located on the opposite side of the main channels. Each vortex channel is a sample with a concave bottom, made along an arc. A single-vane impeller contains a combination of circular and radial channels.

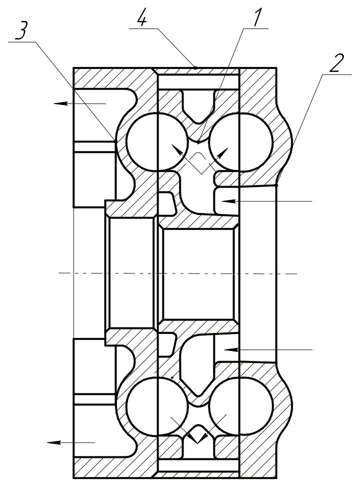


**Fig. 3.** Design of a centrifugal-vortex stage.

To conduct an experiment with this stage, the vortex stage was muffled. The fluid flows through an axial inlet into a single-vane centrifugal wheel, in which it acquires kinetic energy under the action of centrifugal forces. At the exit of the impeller, the liquid enters two ring outlets located from the front and rear face of wheel.

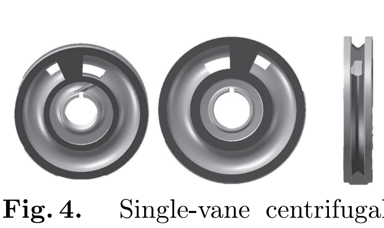

**Fig. 4.** Single-vane centrifugal impeller.

The walls of the outlet are formed by a semicircular channel in the rotating impeller and a semicircular channel in the fixed casing, after which the liquid enters the annular rotating collector through the rectangular window in the semicircular channel of the impeller, from which through the fixed channels in the rectifier located at the rear side of the case falls into the outlet.

## 3. Calculation of centrifugal pump with single-vane impeller

Scientific materials on the pumping of liquids by dynamic pumps with an impeller having a small number of blades (1, 2) are presented in the form of review articles or articles of an advertising nature. Calculation of the parameters of a centrifugal pump with a single-vane impeller is carried out on the basis of the methodology proposed by S. M. Yakhnenko [10], taking into account the ring bend and the end gap between the impeller and the bend. Impeller feed:

$$Q_{im} = \frac{Q}{\eta_v}, \quad \mathrm{m^3/s}, \tag{2}$$

where $Q$ is the pump feed, $\mathrm{m^3/s}$; $\eta_v$ is the volumetric efficiency, dependent on the mechanical seal, $\eta_v = 0.78$ [10]. The calculation of the pump feed depending on the unit diameter $D_q$ is carried out according to the method of S. S. Rudnev [11]

$$Q = D_q^3 n, \quad \mathrm{m^3/s}, \tag{3}$$

where $n$ is the frequency of rotation, rpm;

$$D_q = \frac{b_2}{0.625 k_{in}}, \quad \mathrm{m}, \tag{4}$$

where $b_2$ is the working wheel width, m; $k_{in}$ is the input coefficient. Considering a single-blade impeller, where the compression of the flow by the blade is negligible, we can take $k_{in}$ to be equal to 3.5.

According to experimental data, the head coefficient was obtained depending on the blade installation angle at the exit of the impeller and the experimental coefficients, by the method of S. M. Yakhnenko [10]:

$$\psi = a + b\beta_2, \tag{5}$$

where $a$ and $b$ are the coefficients determined experimentally, $a = 2.2$, $b = 0.02$; $\beta_2$ is the blade exit angle, deg. In another way the head coefficient can be found from the formula:

$$\psi = \frac{2gH}{u_2^2}, \tag{6}$$

where $g$ is the acceleration of gravity, 9.81 $\mathrm{m/s^2}$. Equating these two formulas, we get the value of the pump head:

$$H = \frac{u_2^2\,(a + b\beta_2)}{2g}, \quad \mathrm{m}, \tag{7}$$

where $u_2$ is the circular flow velocity at the exit of the impeller that is determined by the formula:

$$u_2 = \frac{\pi n D_2}{60}, \tag{8}$$

where $D_2$ is the impeller diameter, m. According to this calculation, which is described above, the results for head, power, and efficiency were obtained depending on the flow rate at the optimum point at different rotation frequency. The results are shown in Table 1.

**Table 1.** Parameters at the optimum point.

| | $Q_T$, $\mathrm{m^3/day}$ | $H_T$, m | $N_T$, W | $\eta_T, \%$ |
|---|---|---|---|---|
| 1000 rpm | 36 | 0.9 | 10 | 33 |
| 2000 rpm | 55 | 3.5 | 55 | 41 |
| 3000 rpm | 84 | 7 | 145 | 43 |

## 4. Analysis of experimental studies of centrifugal pump with single-vane impeller

The purpose of the experimental studies was to obtain new information, that is, a set of performance characteristics of a centrifugal pump with a single-vane impeller, taken during its operation on water. Further, only general provisions are noted in determining the quantities necessary to determine the experimental dependencies $H = f(Q)$, $N = f(Q)$, $\eta = f(Q)$.

The feed is determined by the formula:

$$Q = \frac{V}{t}, \quad \text{m}^3/\text{s}, \tag{9}$$

where $V$ is the experimentally measured volume of the working fluid, $\text{m}^3$; $t$ is the measuring tank filling time, s.

The head of the experimental pump is calculated according to the readings of manometers, the measured pressure in the inlet and outlet measuring sections:

$$H = \left[\left(\frac{P_d P_2}{n_{2m}} - \frac{P_s P_1}{n_{1m}}\right) \times 10^4 \rho^{-1} + \frac{u_d^2 - u_s^2}{2g}\right], \tag{10}$$

where $P_s$, $P_d$ are the indications of pressure manometers in the suction and discharge pipelines, div.; $P_1$, $P_2$ are the limiting measured pressures of manometers in the suction and discharge pipelines, respectively, MPa; $n_{1m}$, $n_{2m}$ are the numbers of divisions of manometers on the suction and discharge pipelines, respectively, div.; $\rho$ is the fluid density, $\text{kg}/\text{m}^3$; $u_s$, $u_d$ are the flow velocities of the working fluid in the suction and discharge pipelines, m/s.

The flow velocity is determined by the formula:

$$u = \frac{4Q}{\pi d^2}, \quad \text{m/s}, \tag{11}$$

where $d$ is the diameter of the measuring section in the suction (discharge) lines, m.

The power on the shaft of the experimental pump is determined using a balancing machine and is calculated by the formula:

$$N = F_p l_p \frac{\pi n}{30}, \quad \text{W}, \tag{12}$$

where $F_p$ is the force on the shoulders of the lever, N; $l_p$ is the lever length of balancing machine, m. Efficiency is determined by the formula:

$$\eta = \frac{981 \rho Q H}{N}, \quad \%. \tag{13}$$

The studies were carried out at rotation frequencies of 1000 … 3000 rpm (with increment of 1000 rpm). Fig. 5 shows graphs for a centrifugal pump with a single-vane impeller for water (density $\rho = 1000\ \text{kg}/\text{m}^3$) and rotation frequencies $n = 1000$, 2000 and 3000 rpm, respectively (1, $H = f(Q)$; 2, $N = f(Q)$; 3, $\eta = f(Q)$). The primary analysis of the obtained characteristics, represented graphically by dependencies $H = f(Q)$, $N = f(Q)$, $\eta = f(Q)$, showed the presence of a decreasing nature of the head and power consumption curves.

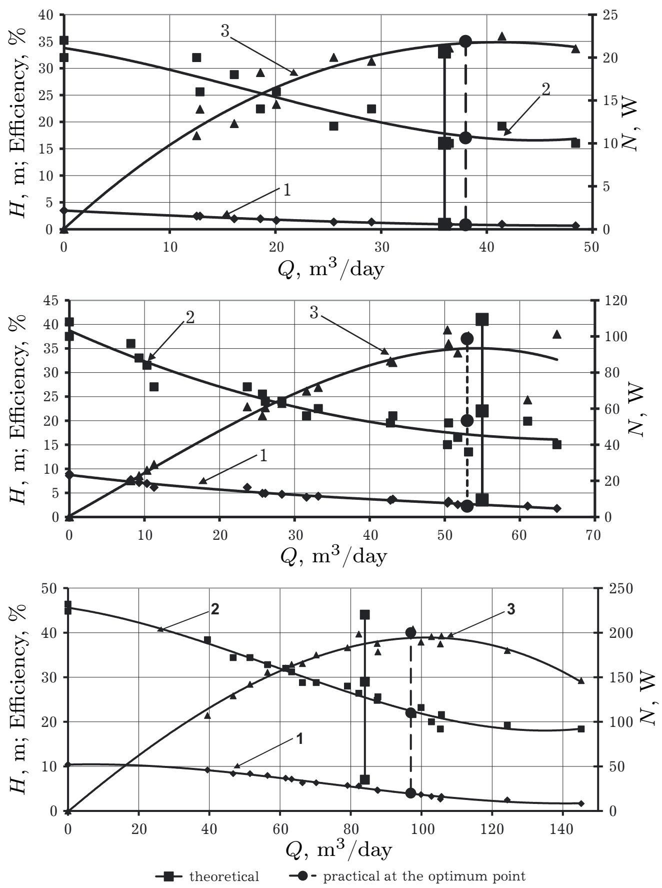


**Fig. 5.** Energy characteristics of a centrifugal pump with a single-vane impeller at $n = 1000$, 2000 and 3000 rpm from top to bottom.

To compare the energy characteristics of the pump, it is convenient to use the ratios of dimensionless head, supply, power, and efficiency factors to the dimensionless coefficients of these parameters at the optimum point when pumping water, namely, at the point of maximum efficiency $\psi/\psi_0$, $\phi/\phi_0$, $\mu/\mu_0$, $\eta/\eta_0$ [12]:

$$\psi = \frac{2gH}{u_2^2}; \quad \phi = \frac{4Q}{\pi D_2^2 u_2}; \quad \mu = \frac{\phi\psi}{\eta} = \frac{8N}{\rho\pi D_2^2 u_2^3 \eta}. \tag{14}$$

Based on the results of the studies, dimensionless coefficients of head ($\psi_0$), feed ($\phi_0$), power ($\mu_0$) and efficiency ($\eta_0$) at the optimum point were obtained and comparative energy characteristics were constructed for rotational frequencies from 1000 to 3000 rpm, which are shown in Fig. 6.

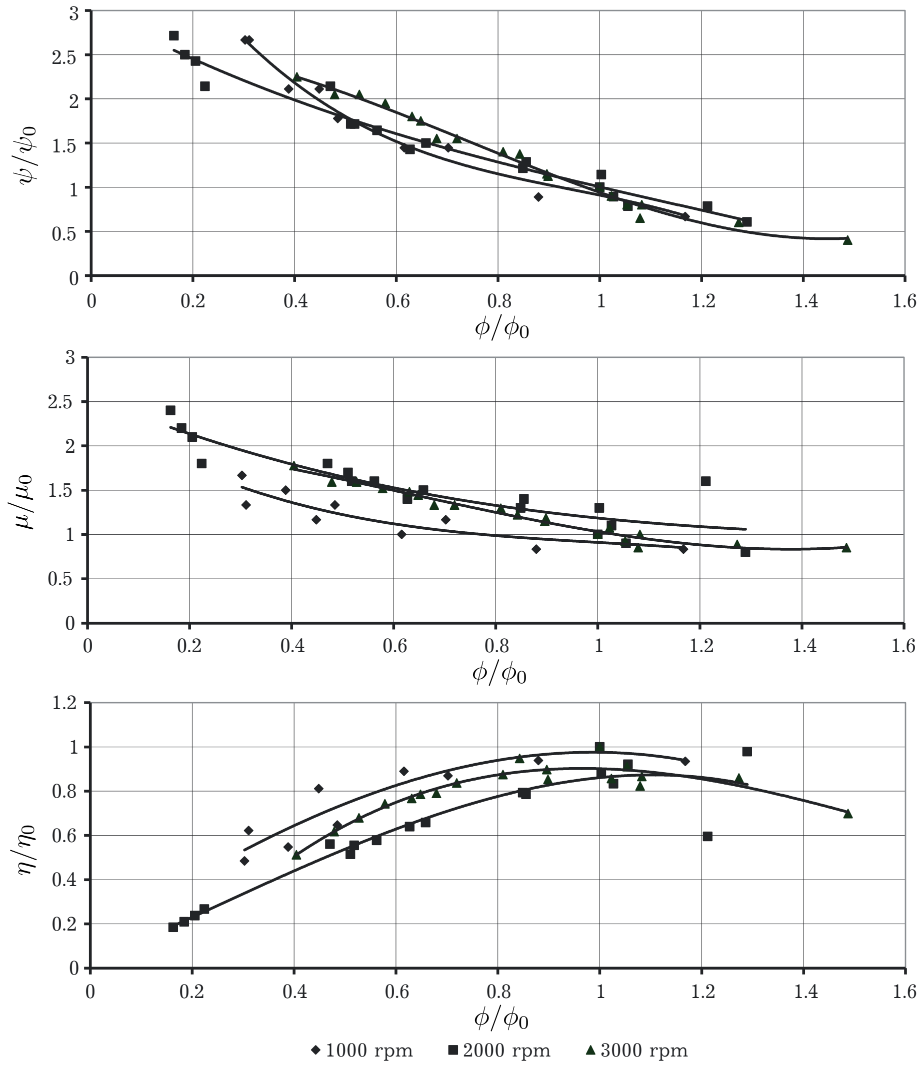


**Fig. 6.** Comparative head, power and efficiency characteristics (from top to bottom) of a pump with a centrifugal-vortex stage.

On the characteristics of head, power and efficiency (Fig. 6), all the curves are closely located near each other. At a frequency of rotation $n = 3000$ rpm, we observe better indicators than at $n = 1000$ rpm and $n = 2000$ rpm. This is due to the fact that with an increase in the frequency of rotation the current around the flow part is improved, hydraulic losses decrease and all this leads to an increase in the energy parameters of the pump [13].

## 5. Conclusions

1. The performance characteristics are obtained of a centrifugal pump with a single-vane impeller of this type at various frequencies of rotation.
2. In the course of the study, it was found that with an increase in the rotation frequency, the parameters of a pump of this design grow. The increase in parameters is explained by the fact that with a growth of the frequency of rotation, the current around the flow part is improved.
3. Comparison of the results obtained during the experiment and when calculating a centrifugal pump with a single-vane impeller shows the difference that is not more than 4%.

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## Математичне дослідження енергетичних характеристик відцентрового насоса з однолопатевим робочим колесом

Трофименко П. Є., Найда М. В., Хоменко О. В.

*Сумський державний університет,*
*вул. Римського-Корсакова, 2, 40007, Суми, Україна*

У статті подано опис конструкції відцентрового насоса з однолопатевим робочим колесом та наведено теоретичний розрахунок такого насоса. Проведено аналіз експериментальних досліджень та порівняння з теоретичними розрахунками. Вперше отримано робочі характеристики насоса з однолопатевим робочим колесом даного виду для різних частот обертання.

**Ключові слова:** *робоче колесо, насос, характеристики рідини, тиск, потік, потужність.*