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New Perspectives on May's Theorem and the Median Voter Theorem

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Abstract

The paper defines and analyzes May's Theorem and the Median Voter Theorem from the Public Policy and Public Choice literature and seeks to compare and contrast the use of both. Through the use of theoretical and applied examples, the paper demonstrates how collective decision-making research has evolved to better inform public policy. Building on Black's (1948) notion that it is the voter in the ideological middle that decides elections, Holcombe (1980) provides an empirical analysis of this theory, Scervini (2012) attempts to show that the middle class (median) voter decides taxation and redistribution policy, Rowley (1984) takes a New Institutional approach to analyzing voters' preferences, Groot and van der Linde (2016) conducts a cross-country analysis to see if the Median Voter Theorem holds true across time and cultures, Carrillo and Castanheira (2008) show that voters change their behavior from the preference of the median voter as the press reveals new information about the quality of candidates which alters voters' perceptions, and Congleton (2003) asserts there may not always be a median voter with examples. Building on May's Theorem that voting is an aggregation of voters' preferences, Hotelling (1929), Black (1948), Maskin (1999), Duggan (2015), and Brady and Chambers (2017) expand on social preference theory showing that Arrow (1951) and May (1952)'s work needed to be updated to include verifiable, empirical tests and further refinements. The paper shows how public policy analysis and group decision making theory and application have evolved over the past 75 years and shines some light on areas for future research and analysis. These findings are important because it will help make candidates and policy proposals more palatable to voters in the ideological middle (median voter) who, as the studies show, often determines the winner. The paper will be of interest to anyone involved in public policy and group decision making processes.

JEL Classification: D70, D72, P48, Z18.

Keywords: May's Theorm, Median Voter Theorem, public policy, public choice, collective decision making, democracy, voting behavior.

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Introduction

This paper will cover analysis of May's Theorem (majority rule) and the Median Voter Theorem (the median voter decides the winner). First, we will define the Median Voter Theorem and explain why it is important to public policy. Second, we will define May's Theorem and explain why it is important to public policy. For both theorems, we will articulate the theorems, the assumptions behind the theorems, and the implications of the theorems. Where appropriate, we will also provide examples to highlight my findings or description. Based on these definitions and explanations, we will compare and contrast them to see how public policy analysis has evolved over the past 75 years. Modern applications are given to some of the latest literature, Alpern and Chen (2021), Heckelman and Ragan (2021), Jones et al. (2021), Miller (2021), and Maskin (2021), on applications of May's Theorem and the Median Voter Theorem. The paper will conclude a comparative analysis of these recent works to foundational works, such as Black (1948), Arrow (1951), May (1952), to see what, if anything, has changed in terms of insight to collective decision-making processes and what they can yield for future policy implications.

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The remainder of the paper is organized as follows: Part II will cover the Median Voter Theorem, Part III will cover May's Theorem, and Part IV will be a conclusion. Attached are works cited.

1. Median Voter Theorem

Black (1948) introduced the Median Voter Theorem, which asserts that the voter in the ideological middle is ultimately the voter, or group of voters, that decides the outcome of any election. Black (1948) distinguishes between the weak and strong variations of the Median Voter Theorem. The weak version asserts that the median voter votes for the winning candidate, or end result in the case of propositions, in every election. The strong version asserts that the median voter determines the winning candidate, or end result in the case of propositions, in every election. This is important because it means that if political parties from polar opposite sides of the political spectrum want their candidate to win, they will have to appeal to the median voter who may have different ideological leanings than the candidate in question. This means that the candidate, particularly a presidential candidate, needs to soften his or her message if he or she wants to win, while at the same time retaining the support of his or her base. The median voter is averse to ideological extremes and prefers a middle-of-the-road approach, even if it is not exactly their ideological preference. Additionally, voters in the middle prefer majoritarian voting methods even if the outcome is not their preference, either. What this means is that voters in the middle care more about the process than the end result. Median voters want egalitarian elections (one person, one vote) even if the winner is not ideologically egalitarian. Again, to middle-of-the-road voters, the election process is more important than the end result.

Holcombe (1980) provides an empirical analysis of the Median Voter Theorem. In his study of 357 educational school districts in Michigan, he distinguishes between the optimal expenditure level for public schools per district compared to the current expenditure level and the preferences of local voters. His study shows that the Median Voter Theorem does prove efficient, at least in the example he provides, for median voters in each school district. This analysis is useful because it shows an example of the Median Voter Theorem in practice. What we will see, however, is that other studies have shown mixed results, and even challenge the Median Voter Theorem.

Other economists have tried similar empirical studies of the Median Voter Theorem. Scervini (2012) sought to verify whether the middle class determined the aggregate level of taxation and redistribution in society; his results not only showed that was not the case, but that the influence of the middle class (median voter) shrunk over time. Building on Downs (1957), Rowley (1984) showed that the Median Voter Theorem was important in showing how the New Institutional approach to analyzing voters' preferences between two competing parties was a better method than the contemporaneous method of spatial theory, which became flawed once certain assumptions were relaxed. Groot and van der Linde (2016) state that among OECD countries, the voter in the 50 percentile often see their preferred taxation rate implemented, although there are differences within and among countries and, using Gini coefficients and the Lorenz curve, demonstrate this is further proof that the Median Voter Theorem holds true. Carrillo and Castanheira (2008) assert that the Black-Downs Median Voter Theorem fails to hold when the press reveals new information about two competing candidates' quality-- that the incomplete information as a result of the press release causes candidates to act in such a way that differs from the preference of the median voter. In other words, the median voter reaches his/her bliss point when either no information or all information about quality is known, but not in-between.

Congleton (2003) notes that there may not always be a median voter. In his example, three voters are trying to decide between three different policy proposals. (Section VIII, p. 7) Their rankings may be intransitive and inconsistent with each other's preferences. The result will be one that pleases one voter with his/her first preference but displeases the other two, being further down their preference scale, but still ranking on their preference scale. What Congleton (2003) fails to mention, however, is that if voters are faced with options, none of which satisfactorily addresses their preferences, they may opt to simply not vote. What we found missing in the literature were the reasons and methods by which voters may boycott the election entirely if none of the candidates or options are sufficiently pleasing to them, no matter how their individual preferences are ranked.

Another example of a case of no median voter would be one in which there are three friends trying to decide on where to go for lunch. In this case, each person wants to go to a different restaurant for lunch. There is no "middle ground" as all three suggestions are quite diverse from each other. Suppose Friend 1 wants Thai, Friend 2 wants Peruvian, and Friend 3 wants Ethiopian. Wherever the three friends decide to go, it will not be a blend of two of the three options (compromise) since they are all unique and cannot be combined; they can only go to one restaurant. What that means is that at least one person has to be willing to forgo his or her

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preference because he or she values the company of the two friends more than his or her preferred eating location. Much like the 'Battle of the Sexes' example in game theory, the same lesson holds true here as well; no matter which candidate or restaurant you prefer, if you want to be with your spouse or companions you should be willing to forgo your preference in order to be with your spouse or companions. The only other recourse in this example is for each friend to go to the restaurant of his/her choice without the other two friends. On net, this will make everyone worse off because while they get the type of food they wanted, they also experience greater diminished utility by not having the company they seek. If they valued the type of food more than the companionship, they would not have sought out the companionship in the first place. So, it behooves each friend to flexible in which restaurant to go to, since they each value the companionship more than the type of food.

2. May's Theorem

May's Theorem refers to the necessary and sufficient conditions necessary for a simple majority decision in voting. May (1952) builds on Arrow (1951)'s assertion that group decision making is an aggregation of individual preferences. May (1952) lays out four conditions regarding majority vote: First, "The group decision function is defined and single valued for every element of U X U X ... X U." This means that all the preferences of the voters are incorporated into the group utility function. (p. 681) Second, "The group decision function is a symmetric function of its argument." (p. 681) This means that all the preferences of the voters match the options available. In other words, all of the voters' preferences are represented in the group utility function. Third, "II: f(-D1, -D2, * . *, Dn) = - f(D1, D2, ... * Dn)." (p. 682). Through the use of a mathematical model, this means that the negative of the accumulated individual preferences is equal to the negative of the function of the accumulated individual preferences. "The third condition is that the method of group decision does not favor either alternative." (p. 681) This means the method of voting has no impact on the result of the vote. In other words, Fourth, "If D = f(D1, D2, , DO) = 0 or 1, and D' = Di for all $i \neq i_0$, and DO > Di$_0$, then D' = f(D', ,D') = 1." (p. 682). May (1952) uses this mathematical model to demonstrate that the next condition is "*positive responsiveness*". That is, the function responds to changes in individual preferences in a positive way, such that if an individual changes his/her preferences positively towards option x, then so too must the group change its collective preference slightly towards option x. Based on these assumptions, May (1952) states his theorem: "A group decision function is the method of simple majority decision if and only if it is always decisive, egalitarian, neutral, and positively responsive." (p. 684) This means that the majority will always win if the voting method is unambiguous in its result, the method is "one person, one vote", if the method of voting is not biased toward any one outcome, and if a positive change in one individual's preference also slightly positively impacts the group's preference. As just discussed, May's Theorem outlines the necessary and sufficient conditions needed for majority voting.

Economists have used empirical methods to test May's Theorem, illuminate some of its implications, and expand upon its assertions. First, Horan et al. (2019) show that May's Theorem does not hold true for three or more candidates and that this contrasts with other studies that have tried to resolve it. Arrow (1963) addresses this issue with the "general possibility theorem", but his analysis is unsatisfactory because it contains a different set of conditions than May's. No generalization can be made between May (1952) and Arrow (1963) that satisfies both sets of conditions for majority rule, even when "Nash independence" (1950) is taken into account. Second, Duggan (2015) shows that within a one-dimensional model of May's Theorem on majority rule, "single peakedness of voter preferences allows us to drop May's restrictive positive responsiveness axiom." (p. 3) In addition, this "result holds when voter preferences are single peaked and linear (no indifferences), in which case a voting rule satisfies anonymity, neutrality, Pareto, and transitivity of weak social preference if and only if the number of individuals is odd and the rule is majority rule." (p. 3) Third, building on Hotelling (1929), Black (1948), Maskin (1999), and Duggan (2015), Brady and Chambers (2017) show that "[i]n a spatial model with Euclidean preferences, we establish that the *geometric median* satisfies Maskin monotonicity, anonymity, and neutrality. For three agents, it is the unique such rule." (p. 657). This is useful because although many political races in the U.S. are between two candidates, an election between three or more candidates is also a likely scenario for which policymakers may prepare.

3. Comparative Analysis

This section will serve as a comparative analysis of the latest literature on these two theorems. In particular, we will examine Alpern and Chen (2021), Heckelman and Ragan (2021), Jones et al. (2021), Miller (2021), and Maskin (2021). In contrast with the previously reviewed literature, these new insights shine new light on how May's Theorem and the Median Voter Theorem can be applied to modern applications.

First, Alpern and Chen (2021) examine the voting procedure of a three-person jury. The jurors are ranked according to ability set in order to achieve the optimal outcome. Assuming that the jurors vote honestly given information signals, the authors state that the median-ability juror should vote first, then highest ability, then lowest ability. Distinguishing between homogeneity and heterogeneity of jurors, Alpern and Chen (2021) show that "for sufficiently homogeneous juries, simultaneous voting is more reliable than sequential voting. On the other hand, when juries are sufficiently heterogeneous, sequential voting is more reliable." This insight is important because it demonstrates that May's Theorem (majority rule) will determine the outcome for homogeneous juries and the Median Voter Theorem (middle voter casts the deciding vote) is more likely to determine the outcome for heterogenous juries. Policymakers would do well to keep this in mind, as jury types can influence juror voting strategies, which impact the outcome of a given case.

Second, Heckelman and Ragan (2021) assert that a Borda count does not need to be limited to a set number of voters and a set number of alternatives. The authors build on May's (1954) conditions that voting be decisive, anonymous, neutral, and positive responsiveness and Young's (1974, 1975, 1995) conditions of decisiveness, neutrality, reinforcement, faithfulness, and cancellation. Heckelman and Ragan (2021) demonstrate "a different set of properties which only Borda satises when both the set of voters and the set of alternatives can vary" and how "Borda is the only scoring rule which will satisfy all of the new properties when the number of voters stays fixed." Through the use of proofs, the authors show that a Borda count, despite initial limitations and weak assumptions, is still the optimal voting mechanism for a situation with two voters and four alternatives. This finding is important because it builds on May's Theorem and the Median Voter Theorem by showing that by changing the voting properties, the limitations of a Borda count can be challenged while retaining the optimal outcome.

Third, Jones et al. (2021) argue that the Median Voter Theorem does not adequately explain the current day phenomenon of massive polarization of the U.S. electorate between the "left" and the "right." The Median Voter Theorem stipulates that when presented with a variety of options, voters in the middle will win the day. This theory is at odds with the modern electorate, which has only dug more deeply into entrenched sides instead of an ideological middle ground. In other words, centrist voters, that is, voters that prefer moderation over one of the extremes, are disappearing. Jones et al. (2021) hypothesize that none of the increasingly extreme options of "right" or "left" are palatable to centrist/moderate voters, causing them to purposefully abstain, something that the Median Voter Theorem cannot explain. Because theory does not adequately match practice, the authors propose three mechanisms to the Median Voter Theorem: 1) "a relative cost of voting that deters voters who are sufficiently indifferent to both candidates," 2) "ideologically motivated third-party alternatives that attract extreme voters," and 3) "a bimodal distribution of voter ideology." These additions are important because they link theory to political reality in an increasingly polarized age.

Next, Miller (2021) extends May's Theorem by introducing a new model of shareholder voting. The author uses ratio rules, difference rules, and share majority rules to show voting strategies used by shareholders to gain a possible advantage over other shareholders. Miller (2021) also introduces a share independence axiom, "which requires that among shareholders with positive shareholdings, the distribution of shares is irrelevant." (p. 13) The author additionally shows how the one share-one vote principle has been challenged to instead reflect the weight of votes based on the proportion of shares a shareholder has. This finding is important, and expands on May's Theorem, because it shows how shareholders may attempt to cheat the system by manipulating their voting authority.

In our final example, Maskin (2021) proposes a modified version of Arrow's (1951) independence of irrelevant alternatives (IIA). The author demonstrates any social choice rule that has the properties of monotonicity and no veto power can be implemented by a game form with three or more individuals. Drawing on Arrow (1951) and May's Theorem, "[R]ather than an impossibility result, [... this criterion] show[s] that a voting rule satisfies modified IIA, Arrow's other conditions, and May's (1952) axioms for majority rule if and only if it is the Borda count (Borda 1781), i.e., rank-order voting." (p. 1) This is significant because it shows weaknesses in and expands upon Arrow's (1951) and May's (1952) conjectures for group-based decisionmaking. The implication of this finding is that the criteria and rules imposed on voting can be re-evaluated to see how the winner is determined.

Conclusion

In conclusion, we have defined and given examples of both May's Theorem (May 1952) and the Median Voter Theorem (Black 1948). Based on Rowley (1984), Congleton (2003), and Groot and Linde (2016), among

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others, we have also discussed some of the implications for both theorems. We have examined how Nash independence and positive responsiveness impact voter behavior and affects different models of voting. In this analysis, we have seen ways in which both theorems are similar (both rely on census and group decision making) and ways in which they are different (the former emphasizes majority rule while the latter emphasizes the ruling of the median voter). We have also shown how public policy analysis and group decision making theory and application has evolved over the last 75 years. Finally, we have shown how both May's Theorem and the Median Voter Theorem have been challenged and defended by further empirical studies (Holcombe 1981, Scervini 2012). Most notably, this analysis is useful because it helps policymakers better understand how to make their proposals more palatable to the median voter, who either votes for the winner (weak model) or determines the winner (strong model), and how to appeal to the majority at large.

Areas for future study include modern applications of the Median Voter Theorem, particularly as it relates to competing voting methods. One could reproduce Holcombe's (1980) and Scervini's (2012) empirical tests in other contexts to see if the same results hold true. Additionally, the role that voting methods has on tax and redistribution policies could be further examined, particularly in the context of current inflation hikes in the U.S. Finally, with many U.S. states experimenting with different voting methods, a cross-state analysis of voting methods and voter turnout would be interesting: Does one voting method significantly increase voter participation while another significantly reduces it? Revisiting May's Theorem and the Median Voter Theorem in these other areas could shine light on voting behavior and outcomes based on different voting systems.

References

1. Black, D. (1948). On the Rationale of Group Decision-Making. *Journal of Political Economy*, *56*(1), University of Chicago Press, 23–34.[Link]

2. Congleton, R. (2003). The Median Voter Model. In Rowley, C. K.; Schneider, F. (eds.). *The Encyclopedia of Public Choice*. Kluwer Academic Press. ISBN 978-0-7923-8607-0. [Google Scholar]

3. Holcombe, Randall G. (1980). An Empirical Test of the Median Voter Model. *Economic Inquiry*. *18*(2), 260–275. [Google Scholar]

4. Scervini, F. (2012). Empirics of the Median Voter: Democracy, Redistribution and the Role of the Middle Class. *Journal of Economic Inequality*, *10*(4), 529–550. [Google Scholar]

5. Rowley, C. K. (1984). The Relevance of the Median Voter Theorem. *Zeitschrift Für Die Gesamte Staatswissenschaft / Journal of Institutional and Theoretical Economics*, 140(1), Mohr Siebeck GmbH & Co. KG, 104–26. [Link]

6. Groot, L., and Daan van der Linde. (2016). Income Inequality, Redistribution and the Position of the Decisive Voter. *Journal of Economic Inequality*, *14*(3), 269–287. [Goole Scholar]

7. Carrillo, J. D., and Castanheira, M. (2008). Information and Strategic Political Polarisation." *Economic Journal*, *118*(530), 845–874. [Google Scholar]

8. May, K. O. (1952). A Set of Independent Necessary and Sufficient Conditions for Simple Majority Decision." *Econometrica*, *20*(4), Wiley, Econometric Society, 680–84. [CrossRef]

9. Arrow, K. J. (1951). Social Choice and Individual Values, *Cowles Commission Monograph* , 12, New York: John Wiley and Sons, 23. [Google Scholar]

10. Arrow, Kenneth J. (1963). Social Choice and Individual Values, 2nd edition (New Haven, CT: Yale University Press. [Link]

11. Horan, S. (2019). Positively Responsive Collective Choice Rules and Majority Rule: A Generalization of May's Theorem to Many Alternatives. *International Economic Review*, *60*(4). [Google Scholar]

12. Nash, J. F. (1950). The Bargaining Problem, *Econometrica*, 18, 155–62. [Google Scholar]

13. Hotelling, Harold. (1990). Stability in Competition, *Spring*, *39*(153), 41–57. [Google Scholar]

14. Maskin, E. (1999). Nash Equilibrium and Welfare Optimality, The *Review of Economic Studies*, *66*(1), 23–38. [Google Scholar]

15. Duggan J. (2015). May's Theorem in One Dimension. *Journal of Theoretical Politics.* [CrossRef]

16. Alpern, S. and Bo, C. (2021). Optimal Voting Order on Sequential Juries: A Median Voter Theorem and Beyond. *Social Choice and Welfare.* [Link]

17. Heckelman, Jac C. and Robi Ragan. (2021). Symmetric Scoring Rules and a New Characterization of the Borda Count. *Economic Inquiry*, *59*(1), 287-299. [Link]

18. Jones, M. I., et al. (2021). Polarization, Abstention, and the Median Voter Theorem. *arXiv preprint arXiv:2103.12847*. Cornell University Press. [Link]

19. Miller, A. D. (2021). Voting in Corporations. *Theoretical Economics*, 16, 101-128. [Link]

20. Maskin, E. (2021). Arrow's Theorem, May's Axioms, and Borda's Rule. *Scholar.harvard.edu* [Link]

21. Young, H.P. (1974). An Axiomatization of Borda's Rule. *Journal of Economic Theory*, *9*(1), 43-52. [Google Scholar]

22. Young, H.P. (1975). Social Choice Scoring Functions. *SIAM Journal of Applied Mathematics*, *28*(4), 824-838. [Google Scholar]

23. Young, P. (1995). Optimal Voting Rules. *Journal of Economic Perspectives*, *9*(1), 51-64. [Link]