JOURNAL OF NANO- AND ELECTRONIC PHYSICS
Vol. **15** No 4, 04017(4pp) (2023)

ЖУРНАЛ НАНО- ТА ЕЛЕКТРОННОЇ ФІЗИКИ
Том **15** № 4, 04017(4cc) (2023)

Homogeneous Barotropic String Cosmology Model of Bianchi Type IV_0 in General Relativity

Sunil Kumawat*, Laxmi Poonia†

Department of Mathematics and Statistics, Manipal University Jaipur, India

(Received 16 June 2023; revised manuscript received 19 August 2023; published online 30 August 2023)

The homogeneous barotropic string cosmology model has been widely investigated in the context of string theory and other quantum gravity theories. It has been shown that it leads to intriguing and non-trivial predictions for the early universe, such as the development of cosmic strings and the generation of gravitational waves. It is based on the Bianchi IV_0 homogeneous generalization of stress-energy-momentum, density, and pressure may be employed to the goal of this study is to find a solution to the Einstein metric field equations. The model is characterised by a collection of non-linear, coupled differential equations that explain the development of the scale factor of the universe, in addition to the energy density and pressure of the string fluid. The model is expanding, not rotating, non-shearing, and has an anisotropic structure. The mathematical underpinnings and physical ramifications of the model of Bianchi Type IV_0 in General Relativity are discussed in this study According to the authors, particle density (p) is less than half of the rest energy density while string tension density is larger than half of the rest energy density. When the cosmic time approaches infinity, the cosmic string disappears and the model transforms into a vacuum universe. The results of this study provide insights into the behavior of a model, offering a deeper understanding of the interplay between different physical and kinematical properties and their evolution over time.

Keywords: Bianchi type VI_0, Space-time, General relativity.

DOI: 10.21272/jnep.15(4).04017 PACS number: 98.80. – k

1. INTRODUCTION

Cosmology is the branch of astronomy. One important area of study in cosmology is the use of string cosmology models to understand the early stages of the universe, galaxy formation, and the accelerating expansion of the universe. These models play a significant role in our understanding of the universe, providing a framework for exploring the early stages of the universe and the formation of galaxies. By continuing to study these models and refine our understanding of the universe, cosmologists hope to gain a deeper understanding of the evolution of the universe and the role of strings in its development. String cosmology models were first proposed by Kibble and Vilenkin [1, 9], who demonstrated how small topological flaws like vacuum domain walls, threads, walls bound by strings, and monopoles coupled by strings might result from phase transitions in the early cosmos. A dark energy model in Bianchi Type VI_0 was discovered by Pradhan et al. [2, 4]. Recent years have seen a number of investigations on the General Relativity Bianchi Type VI_0 cosmological model in a different field, Bali and Poonia [5], with a time-dependent equation of state, Tyagi et al. with a cosmological model for anti-stiff perfect fluid [6-7]. We may learn about the large-scale dynamics of the universe, including the rate of expansion and the curvature of spacetime, by studying the homogeneous barotropic string cosmology model of Bianchi Type IV_0 in General Relativity. It also sheds light on how the ideal fluid, which makes up the majority of cosmological theories, behaves as the universe expands. It is thought that the entire cosmos is made up of a perfect fluid, whose pressure and energy density are related by a barotropic equation of state. The results of this study provide insights into the behavior of a Bianchi type VI_0 string cosmological model, offering a deeper understanding of the interplay between different physical variables and their evolution over time. The model offers an interesting alternative to traditional models and could provide a new framework for further research in the field of cosmology.

2. SOLUTION OF FIELD EQUATIONS

We think about Bianchi type VI_0 inhomogeneous metric.

$$ds^2 = -dt^2 + p^2(t)dx^2 + e^{2x}q^2(t)dy^2 + e^{-2x}r^2(t)dz^2 \quad (1)$$

The Einstein field equation for a cloud string takes the form

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j \quad (2)$$

and

$$T_i^j = \rho v_i v^j - \lambda x_i x^j \quad (3)$$

Where Unit flow vector is v_i with satisfying the condition $v^i.v_i = 1, x^i.x_i = -1$ and $v^i.x_i = 0$.

Here, rest-energy (ρ) and rest energy density (ρ_p) are $\rho = \rho_p + \lambda$

Where λ denote the density of tension in the string.

* sunilkumawatsir@gmail.com

† poonia@jaipur.manipal.edu

The results were presented at the 3rd International Conference on Innovative Research in Renewable Energy Technologies (IRRET-2023)

Considering the coordinates as

$$v^i = \left(0,0,0,\frac{1}{p(x,t)}\right)$$

The equidistant point (x^i)should be chosen so that it is equidistant to the *x*-axis

$$x^i = \left(\frac{1}{p(x,t)},0,0,0\right)$$

A line-element (2) and (3) are induced by Einstein's field equation (1) for which the following equation can be derived

$$\frac{q_{44}(t)}{q(t)} + \frac{r_{44}(t)}{r(t)} + \frac{q_4(t)r_4(t)}{q(t)r(t)} + \frac{1}{p^2(t)} = -\lambda \tag{4}$$

$$\frac{p_{44}(t)}{p(t)} + \frac{r_{44}(t)}{r(t)} + \frac{p_4(t)r_4(t)}{p(t)r(t)} - \frac{1}{p^2(t)} = 0 \tag{5}$$

$$\frac{p_{44}(t)}{p(t)} + \frac{q_{44}(t)}{q(t)} + \frac{p_4(t)q_4(t)}{p(t)q(t)} - \frac{1}{p^2(t)} = 0 \tag{6}$$

$$\frac{p_4(t)}{p(t)}\left(\frac{q_4(t)}{q(t)} + \frac{r_4(t)}{r(t)}\right) + \frac{q_4(t)r_4(t)}{q(t)r(t)} - \frac{1}{p^2(t)} = \rho \tag{7}$$

$$\frac{q_4(t)}{q(t)} - \frac{r_4(t)}{r(t)} = 0 \tag{8}$$

The suffices 4 after *p*, *q*, and *r* denote partial differentiation concerning t respectively
By equation (8):

$$\frac{q_4(t)}{q(t)} = \frac{r_4(t)}{r(t)} \tag{9}$$

By integration

$$q(t) = k\,r(t) \tag{10}$$

From equations (5) and (6) we have

$$\frac{q_{44}(t)}{q(t)} = \frac{r_{44}(t)}{r(t)} \tag{11}$$

Let $p(t) = q^n(t)$

Then
$$\frac{p_4(t)}{p(t)} = n\frac{q_4(t)}{q(t)} \tag{12}$$

From equation (4)

$$-2\frac{q_{44}(t)}{q(t)} - \left(\frac{q_4(t)}{q(t)}\right)^2 - \frac{1}{q^{2n}(t)} = \lambda \tag{13}$$

From eq (7)

$$(2n+1)\left(\frac{q_4(t)}{q(t)}\right)^2 - \frac{1}{q^{2n}(t)} = \rho \tag{14}$$

Let
$$\lambda = \gamma\rho \tag{15}$$

$$\frac{q_{44}(t)}{q(t)} + \left(\frac{1+\gamma(2n+1)}{2}\right)\left(\frac{q_4(t)}{q(t)}\right)^2 = \left(\frac{1+\gamma}{2}\right)\frac{1}{q^{2n}(t)}$$

Let
$$k_1 = \frac{1+\gamma(2n+1)}{2} \ \ \& \ \ k_2 = \frac{1+\gamma}{2}$$

$$\frac{q_{44}(t)}{q(t)} + k_1\left(\frac{q_4(t)}{q(t)}\right)^2 = k_2\frac{1}{q^{2n}(t)} \tag{16}$$

$$\frac{d}{dt}\left(q^{k_1}(t)q_4(t)\right) = k_2 q^{k_1+1-2n}(t) \tag{17}$$

Integrate both sides

$$q^{k_1}(t)q_4(t) = k_2\frac{q^{k_1+2-2n}(t)}{k_1+2-2n} + c_1 \tag{18}$$

Let
$$k_3 = \frac{k_2}{k_1+2-2n} \ \ \& \ \ c_1 = 0$$

$$q_4(t) = k_3\, q^{2-2n}(t)$$

Integrate both sides

$$q(t) = \left((2n-1)k_3t + C\right)^{\frac{1}{2n-1}} \tag{19}$$

$$r(t) = k\left((2n-1)k_3t + C\right)^{\frac{1}{2n-1}} \tag{20}$$

$$p(t) = \left((2n-1)k_3t + C\right)^{\frac{n}{2n-1}} \tag{21}$$

By using equation (1) with (19), (20), & (21)

$$\begin{aligned} ds^2 = &-dt^2 + \left((2n-1)k_3t + C\right)^{\frac{2n}{2n-1}} dx^2 \\ &+ e^{2x}\left((2n-1)k_3t + C\right)^{\frac{1}{2n-1}} dy^2 \\ &+ e^{-2x}k\left((2n-1)k_3t + C\right)^{\frac{1}{2n-1}} dz^2 \end{aligned} \tag{22}$$

3. SOME PHYSICAL PROPERTIES

The following properties are determined by the mathematical solutions to our model, which describe the behavior of space-time in the presence of other physical quantitate.

The universe is expanding in all three spatial directions, but at different rates, resulting in a highly anisotropic universe

$$\theta = \frac{(n+2)k_3}{(2n-1)k_3t + C} \tag{23}$$

Share

$$\sigma = \frac{(n-1)}{\sqrt{3}}\left(\frac{q_4(t)}{q(t)}\right) = \frac{(n-1)k_3}{\sqrt{3}\left[(2n-1)k_3t + C\right]} \tag{24}$$

Spatial volume

$$V = kk_3\left((2n-1)k_3t + C\right)^{\frac{n+2}{2n-1}} \tag{25}$$

Hubble parameter

$$H = \frac{(n+2)k_3}{3\left((2n-1)k_3t + C\right)} \tag{26}$$

The universe is increase with an accelerating rate in the Bianchi type VI0 model.

$$q = -1 - \frac{n+2}{3}\left[\frac{k_3^2(2n-1)}{[(2n-1)k_3t + C]^2}\right] \tag{27}$$

string tension density

$$\lambda = \frac{q_{44}(t)}{q(t)} + \frac{r_{44}(t)}{r(t)} + \frac{q_4(t)r_4(t)}{q(t)r(t)} + \frac{1}{p^2(t)}$$

$$\lambda = \left(\frac{2k_2+1}{\left((2n-1)k_3t + C\right)^{\frac{2n}{2n-1}}}\right) + (1-k_1)\left(\frac{k_3}{(2n-1)k_3t + C}\right)^2 \tag{28}$$

energy density

$$\rho = \frac{p_4(t)}{p(t)}\left(\frac{q_4(t)}{q(t)} + \frac{r_4(t)}{r(t)}\right) + \frac{q_4(t)r_4(t)}{q(t)r(t)} - \frac{1}{p^2(t)}$$

$$\rho = (2n+1)\left(\frac{k_3}{(2n-1)k_3t + C}\right)^2 - \frac{1}{\left((2n-1)k_3t + C\right)^{\frac{2n}{2n-1}}} \tag{29}$$

The ratio of string tension density and energy density

$$\frac{\lambda}{\rho} = \frac{\left(\frac{2k_2+1}{\left((2n-1)k_3t + C\right)^{\frac{2n}{2n-1}}}\right) + (1-k_1)\left(\frac{k_3}{(2n-1)k_3t + C}\right)^2}{(2n+1)\left(\frac{k_3}{(2n-1)k_3t + C}\right)^2 - \frac{1}{\left((2n-1)k_3t + C\right)^{\frac{2n}{2n-1}}}} \tag{30}$$

$$\lim_{t\to\infty}\left(\frac{\lambda}{\rho}\right) \neq 0 \tag{31}$$

It was found that the particle density also differs from the rest energy density eventually when the time approaches infinity.

Graphical representation of physical and geometrical parameters

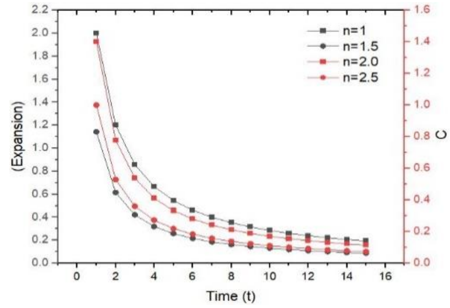

Fig. 1 – Graph Between time and expansion

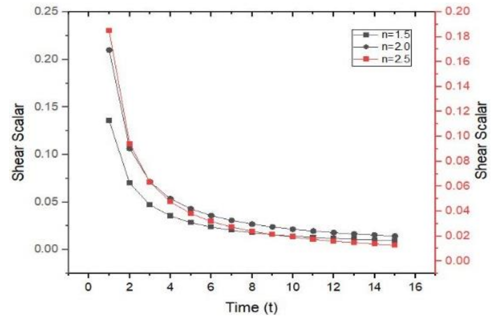

Fig. 2 – Graph Between time and Shear scalar

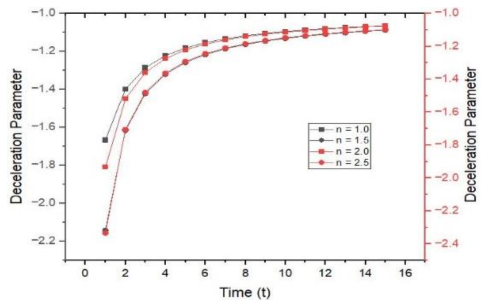

Fig. 3 – Graph Between time & Deceleration Parameter

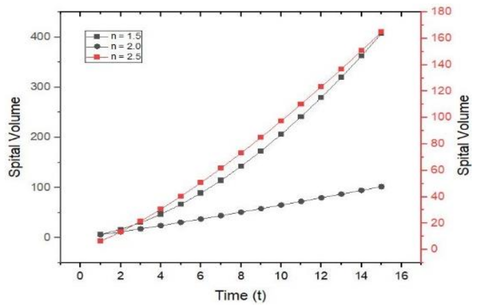

Fig. 4 – Graph Between time and Spital Volume

4. CONCLUSION

In this study, a Bianchi type VI_0 string cosmological model is introduced which is homogeneous in space and has several unique characteristics. The model is expanding, non-shearing, not rotating, and has an anisotropic structure. Additionally, the model does not have an initial singularity, which makes it even more interesting. We investigated and verify with the graph between θ, σ, and H decrease with time (cosmic), and volume (V) increases with time. It was found that the ratio of string tension density (λ) and the rest energy density is great than one and the ratio of the particle density and rest energy density is less than half. Over time, both λ, ρ, and ρ_p decrease, and eventually when the time approaches infinity, the cosmic string disappears, and the model becomes a vacuum universe.

REFERENCES

1. A. Vilenkin, *Phys. Rep.* **121** No 5, 263 (1985).
2. A. Pradhan, R. Jaiswal, K. Jotania, R.K. Khare, *Astrophys. Space Sci.* **337** No 1, 401 (2012).
3. A. Tyagi, R.B. Tripathi, *Prespacetime J.* **8** No 7, 254 (2017).
4. M.K. Verma, S. Ram, *Appl. Math.* **2** No 3, 348 (2011).
5. R. Bali, L. Poonia, *Int. J. Modern Phys. World Sci. Pub. Commun. Conf. Ser.* **22**, 593 (2013).
6. R. Bali, P. Kumari, L. Zanetti, H. Çāglar, S. Aygün, H. Shanjit, S. Nerkar, *Adv. Astroph.* **2** No 2, 67 (2017).
7. S.R. Bhoyar, V.R. Chirde, S.H. Shaikh, *Prespacetime J.* **8**, 9 (2017).
8. S. Parikh, A. Tyagi, B.R. Tripathi, *Prespacetime J.* **8** No 4, 352 (2017).
9. T.W. Kibble, *J. Phys. A: Math. General* **9** No 8, 1387 (1976).
10. T.W.B. Kibble, *J. Phys. A, Math. Gen.* **9**, 1387 (1976).
11. T.W.B. Kibble, *Phys. Rep.* **67**, 183 (1980).
12. U.V. M. Rao, et al., *Astrophys. Space Sci.* **314**, 73 (2008).

Космологічна модель гомогенної баротропної струни Б'янкі типу IV_0 в загальній теорії відносності

Sunil Kumawat, Laxmi Poonia

Department of Mathematics and Statistics, Manipal University Jaipur, India

Космологічна модель однорідної баротропної струни була широко досліджена в контексті теорії струн та інших теорій квантової гравітації. Було показано, що це веде до інтригуючих і нетривіальних прогнозів для раннього Всесвіту, таких як розвиток космічних струн і генерація гравітаційних хвиль. Модель заснована на однорідному узагальненні напруги-енергії-імпульсу, щільності та тиску Біанкі IV_0, який може бути використаний для досягнення мети цього дослідження – розв'язку рівнянь метричного поля Ейнштейна. Модель характеризується набором нелінійних диференціальних рівнянь, які пояснюють розвиток масштабного фактора Всесвіту, на додаток до щільності енергії та тиску рідини струни. Модель розширюється, не обертається, не зсувається і має анізотропну структуру. У цьому дослідженні обговорюються математичні основи та фізичні розгалуження моделі Б'янкі типу IV_0 у загальній теорії відносності. Відповідно до авторів, щільність частинок (p) становить менше половини щільності енергії спокою, тоді як щільність натягу струни перевищує половину щільності енергії спокою. щільність енергії спокою. Коли космічний час наближається до нескінченності, космічна струна зникає, а модель перетворюється на вакуумний Всесвіт. Результати цього дослідження дають змогу зрозуміти поведінку моделі, пропонуючи глибше зрозуміти взаємодію між різними фізичними та кінематичними властивостями та їх еволюцію з часом.

Ключові слова: Б'янкі тип VI_0, Простір-час, Загальна теорія відносності.