J. Nano- Electron. Phys.
1 (2009) No3, P. 43-51

PACS numbers: 72.15.Jf, 72.20.Pa, 85.80. – b

PHYSICAL PROCESSES IN ANISOTROPIC THERMOELEMENT AND THEIR FEATURES

V.M. Matyega, O.G. Danalakiy

National Technical University "Kharkov Polytechnic Institute",
Chernivtsi Department,
203-a, Holovna Str., 58018, Chernivtsi, Ukraine
E-mail: OGDanalaki@gmail.com

We investigate the galvano-thermomagnetic cooling elements of the transverse-type and anisotropic thermoelectric cooling elements with and without a cooling substrate, the hollow circular-cylindrical galvano-thermomagnetic cooling elements and anisotropic cooling elements, the operating principle of which is based on the idea of direct contact between the "hot" face of a cooling element and the coolant of a thermostat.

Keywords: *THERMOELEMENTS, PHYSICAL PROCESSES, TEMPERATURE DISTRIBUTION, THERMAL EMF.*

(Received 24 July 2009, in final form 27 November 2009)

1. INTRODUCTION

The present work is focused on the study of the nature of physical processes in thermoelectric mediums, which are the basis of discovery of original and important from the practical point of view elements and investigation of the phenomena in these objects. Questions connected with cascading of the transverse galvano-thermomagnetic cooling elements (GTM CE) and anisotropic thermoelectric cooling elements (AT CE) are poorly covered. This idea is based on the assumptions about the one-dimensionality of the temperature and the constancy of the electric field, which cannot be held simultaneously. Presented results of the physical investigations, directed to elucidate these questions, are the components of thermoelectricity and have the original nature from the point of view both of the phenomena and the practical application. The aim of the present paper is to develop the models of new longitudinal and transverse anisotropic and GTM elements and investigation of the physical processes in these objects; clarification of the improvement possibilities of the operating characteristics of usual longitudinal thermal elements by their modernization; elucidation of the nature of the physical processes in thermal elements with side heat-exchange and in loaded anisotropic thermal elements (ATE).

To achieve the desired aim it is necessary to solve the following problems:
1. Analyze operation of the two-stage AT CE based on the one-dimensional temperature model with the assumption of the constancy of the electric current density from the point of view of the maximum temperature drop, and generalize the results obtained for the case of the model of multistage transverse galvano-thermomagnetic cooler (GTMC) and anisotropic thermoelectric cooler (ATC).

2. Analyze the influence of anisotropy of the thermal conductivity on the transverse thermal emf of ATE under condition that the anisotropy parameter is much more than 1.
3. Investigate the vortex anisotropic thermal elements and physical processes in these objects.

To achieve the aim and to solve the aforesaid problems we have used the fundamental concepts of thermodynamics of the thermoelectric phenomena and methods of the mathematical physics.

In the present work we have studied the physical processes in ATE from the point of view of the one- and two-dimensional temperature models, the use validity of which was proved in [3]. One-dimensional temperature model is used while investigating the operation of cascade ATC, and two-dimensional one – while investigating the influence of the anisotropy of thermal conductivity on the temperature field of ATE.

Operation of the loaded ATE from the point of view of the conversion efficiency is studied.

2. ANISOTROPIC THERMAL ELEMENT IN COOLING MODE

Schematic diagram of the AT CE is represented in Fig. 1. In this paper we present the investigation results first published in [1].

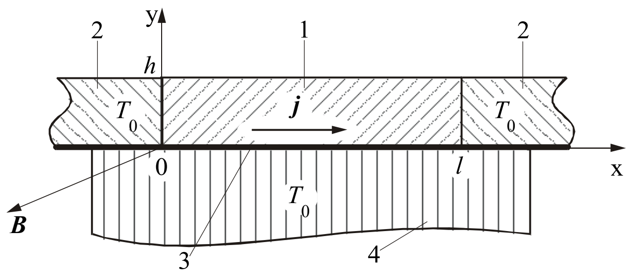

***Fig. 1** – Schematic representation of AT CE or GTM CE if sample 1 is in magnetic field perpendicular to the xy-plane. Current leads 2 are made of metal (for example, copper) and contact with the thermostat 4 through the thin dielectric layer 3 with high thermal conductivity*

If cooling element (CE) is long enough one can consider the temperature as $T = T(y)$ in its midsection. For the electric field component along the x-axis we can write

$$E_1 = \rho j + \alpha_{12}\partial T/\partial y,$$

where ρ and α_{12} are the longitudinal resistivity and the coefficient of the transverse thermal emf, respectively, which are considered to be constant. In accordance with the continuity equation $j = j(y)$.

Along the y-axis the electric field conditioned by a temperature gradient also appears:

$$E_2 = \alpha_{22}\partial T/\partial y,$$

where α_{22} is the coefficient of the thermal emf along the y-axis, which is also constant. Electric field should be the potential one, i.e.,

$$\frac{\partial E_1}{\partial y} = \frac{\partial E_2}{\partial x},$$

therefore $E_1 = \text{const}$ since E_2 does not depend on x. So, we conclude: if temperature is one-dimensional than $E_1 = \text{const}$. In this approximation the energy conservation law can be written in the form

$$(1 + ZT)\frac{d^2T}{dy^2} + Z\left(\frac{E_1}{\alpha_{12}} - \frac{dT}{dy}\right)^2 = 0\,, \tag{1}$$

where $Z = \alpha_{12}^2 / (\chi\rho)$ is the anisotropic thermoelectric efficiency, ρ and χ are the resistivity and the thermal conductivity along the x- and y-axes, respectively.

Equation (1) should be considered jointly with the boundary conditions:

$$\begin{aligned} T(0) &= T_0, \\ T(h) &= T_h. \end{aligned} \tag{2}$$

General solution of equation (1) has a form

$$\frac{1}{1 - ZT + Z(E_1 / \alpha_{12})y - A} + \ln(1 - ZT + (E_1 / \alpha_{12})y - A) = B\,, \tag{3}$$

where A and B are the constants of integration, which are found from the boundary conditions (2). We will assume that the top face of AT CE (Fig. 2) is adiabatically isolated from the environment. This can be achieved if CE, for example, is placed in the vacuum. Condition of the adiabatic isolation is the following:

$$-\chi\left.\frac{\partial T}{\partial y}\right|_{y=h} + \alpha_{12} j T_h = 0\,. \tag{4}$$

Using conditions (2) and (4) and expression (3) we will obtain the equation

$$\frac{1 + ZT_0}{1 + ZT_0 + ZT_h - z(E_1 / \alpha_{12})h} + \ln(1 + ZT_0 + ZT_h - Z(E_1 / \alpha_{12})h) = 1 + ZT_h\,, \tag{5}$$

which connects T_h and E_1. In general case it does not have an analytical solution with respect to T_h. In the case when $Z(T_0 - T_h) << 1$ we find from (5)

$$T_h = T_0 - \frac{1}{Z}\left[1 + ZT_0 - \frac{1 + ZT_0}{1 - Z(E_1 / \alpha_{12})h} - \ln\left(1 - Z\left(E_1 / \alpha_{12}\right)h\right)\right].$$

In this case the optimal value of the parameter $C = Z(E_1/\alpha_{12})/h$ is equal to $C = ZT_0$, therefore the minimum temperature will be the following:

$$T_h = T_{\min} = \frac{\ln(1 + ZT_0)}{Z}\,.$$

If $ZT_0 << 1$ than $T_{\min} = T_0 - ZT_0^2 / 2$.

Approximation j = const for the one-dimensional temperature distribution leads to the following expression for T_{min} [3]:

$$T_{min} = \frac{\sqrt{1 + ZT_0} - 1}{Z},$$

which holds for any Z. For small Z when $2ZT_0 << 1$ we obtain $T_{min} = T_0 - ZT_0^2 / 2$. In [3] approximation j = const is preferred, since it can be easily realized experimentally.

3. STEADY-STATE TEMPERATURE REGIME OF THE TWO-STAGE ANISOTROPIC THERMAL ELEMENT IN COOLING MODE

It was mentioned above, that investigations of cascading of the transverse coolers [4, 5] are not convincing enough: physical models of separate and cascading coolers, which are the basis of these investigations, are too far from the reality. Separate galvano-thermomagnetic (GTM) or anisotropic thermoelectric (AT) elements of the squared shape are placed one above the other. Here we suppose that the heat extracted by each element is the thermal load of thermal elements. While calculating the maximum temperature decrease a number of assumptions should be made, and the main ones are the following: the same electrostatic field is applied to each CE; there is an electric contact between the separate thermal elements, and as it seems, it does not influence the current and temperature distributions.

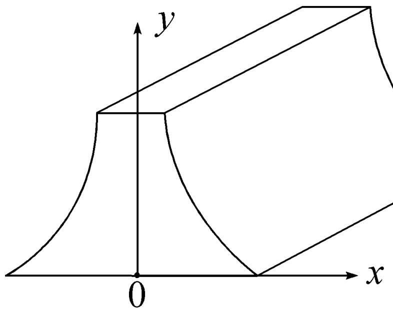

***Fig. 2** – Cascade transverse GTM or AT element operating in cooling mode*

Calculations performed on the basis of the mentioned simplifications lead to that the maximum temperature decrease is achieved by the exponential change in the side generatrixes subject to x (Fig. 2). Experiments with GTMC showed that the exponential cross-section gives deeper cooling than the same GTMC of the squared shape. In [5] the investigation results of cascade transverse GTMC are generalized for the case of ATC.

In this paper the two-stage ATC from the point of view of the maximum temperature [6] is studied in detail. Schematic diagram of the two-stage ATC is represented in Fig. 3. It consists of the separate AT CE 1 and AT CE 2, which are connected in the thermal ratio in such a way that the heat extracted on the bottom face of AT CE 2 is the thermal load of AT CE 1. There is no electric contact between the bottom face of AT CE 2 and the top face of AT CE 1. At the same time thermal contact between them is considered to be the perfect one. In electric ratio the AT CE 1 and AT CE 2 are connected as it has shown in Fig. 3.

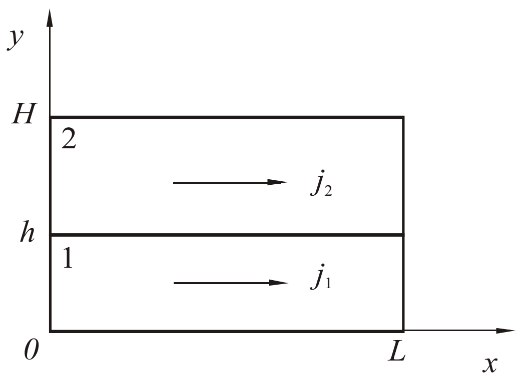

Fig. 3 *– Schematic diagram of the two-stage ATC*

The model described here differs from the one presented in [5] by the fact that, firstly, instead of the condition of the constancy of the electric field the condition j = const is taken. At first sight the condition E = const is stricter. However, it is difficult to realize this condition. Condition j = const is satisfied easier: for this it is necessary to produce the corresponding current leads [3]. Secondly, cascading, which is known from [5], foresees the identity of the cooling coefficients of the separate thermal elements that is beneath criticism as well.

Choosing AT CE 1 and AT CE 2 long enough, i.e., assuming $H/L << 1$ and $l/L << 1$, where h and $l = H - h$ are the sizes of CE 1 and CE 2 along the y-axis, L is the size along the x-axis (Fig. 4), one can consider that in the midsection of ATE the temperature will depend on the y only. We also suppose that the kinetic coefficients of materials of ATE 1 and ATE 2 do not depend on the temperature. This assumption is confirmed if the operating temperature range is not very wide.

Generalized equation of the thermal conductivity in the steady-state case under condition that the current densities in CE 1 and CE 2 are constant will be written in the form

$$\frac{d^2T_i}{dy^2} + b_i = 0\,, \tag{5}$$

where $i = 1, 2$ is a number of CE, $b_i = \rho_i j_i^2 / \chi_i$.

Boundary conditions are the following:

$$T_1(0) = T_0,\ T_1(h) = T_h,\ T_2(h) = T_h,\ T_2(H) = T_H. \tag{6}$$

General solution of equation (5) has the form

$$T_i\left(y\right) = -\frac{1}{2}b_i y^2 + A_i y + B_i\,, \tag{7}$$

where A_i and B_i are the constants of integration. Using the boundary conditions (6) and solution (7) we can find the expressions for the temperature distributions in CE

$$T_1(y) = T_0 - \frac{1}{2}b_1 y^2 + \left(\frac{1}{2}b_1 h - \frac{T_0 - T_h}{h}\right)y\,,$$

$$T_2(y) = T_h - \frac{1}{2}b_2 y^2 + \left(\frac{1}{2}(H+h) - \frac{T_h - T_H}{l}\right)y - \frac{1}{2}b_2 Hh - \frac{T_h - T_H}{l}h\,.$$

We will obtain temperatures T_h and T_H from the conditions

$$-\frac{1}{2}b_1 h^2 - T_0 + T_H - a_1 h T_h = k\left(\frac{1}{2}b_1 l^2 - T_h + T_H - a_2 l T_h\right),$$

$$\frac{1}{2}b_2 + T_h - T_H + a_2 l T_H = 0\,,$$

where $a_i = \dfrac{\alpha_i j_i}{\chi_i}$, $k = \dfrac{\chi_2 h}{\chi_1 l}$, α_i is the transverse thermal emf.

These conditions indicate, respectively, the continuity of the heat flow at the interface between AT CE 1 and AT CE 2 and the adiabatic isolation of the top face of AT CE 2. Using them we find

$$T_H = \frac{1}{1 - a_2 l}\left(T_h + \frac{1}{2}b_2 l^2\right), \tag{8}$$

$$T_h = \frac{\frac{1}{2}kb_2 l^2 + (1 - a_2 l)\left(T_0 + \frac{1}{2}kb_2 l^2 + \frac{1}{2}b_1 h^2\right)}{1 - a_1 h(1 - a_2 l) - a_2 l(1 + k a_2 l)}\,. \tag{9}$$

Let us consider two cases.

1) $a_1 > 0$, $a_2 > 0$.

In order that cooling takes place, i.e., $T_H < T_0$, it is necessary to suggest in (8) and (9) that $j_1 < 0$ and $j_2 < 0$, that is to direct currents in the negative direction of the x-axis. In this case we can state about the "parallel" connection of AT CE 1 and AT CE 2. Let us suppose that $k << 1$, $\alpha_1 = \alpha_2 = \alpha$, $\rho_1 = \rho_2 = \rho$ and $\chi_1 = \chi_2 = \chi$. Then we will obtain for T_H

$$T_H = \frac{T_0 + \frac{1}{2}\rho I_1^2/(\chi c^2)}{1 + \alpha I_1/(\chi c) + 2\alpha I_2/(\chi c)} + \frac{\frac{1}{2}\rho I_2^2/(\chi c^2)}{1 + \alpha I_1^2/(\chi c)}\,,$$

where I_1 and I_2 are the currents in AT CE 1 and AT CE 2, respectively, c is the CE thickness. In the last expression we neglect the quantities of the second order of smallness $\alpha I_1/(\chi c)$, $\alpha I_2/(\chi c)$ and $[\alpha I_2/(\chi c)^2]k$. Further investigations are to fill in the optimal currents I_1 and I_2, which, generally speaking, should be different. For numerical estimation we take $I_1 = I_2 = I$. Let $\alpha = 10^{-4}$ V/K, $\chi = 10^{-2}$ W/(cm·K), $\rho = 10^{-3}$ Ohm·cm, $c = 1$ cm, $I = 10$ A, $T_0 = 300$ K, then find $T_H = 239$ K. For one-stage AT CE in accordance with formula

$$T_{\min} = \frac{\sqrt{1 + ZT_0} - 1}{Z},$$

where $Z = \alpha^2/(\chi\rho)$ is the anisotropic thermoelectric Q-factor. At $Z = 10^{-3}$ K^{-1} we have $T_{\min} = 265$ K. So, cooling is deeper for the two-stage ATC.

2) $a_1 < 0$, $a_2 > 0$.

In this case it is necessary to change the current direction in AT CE 1 by the opposite one. It is appropriate mention here about the "series" connection of AT CE 1 and AT CE 2. Taking the same values of the kinetic coefficients, sizes and currents as in the first case, we will obtain $T_H \approx 239$ K.

We should note, that the given calculations have only to show that in the case of ATC cascading leads to deeper cooling.

4. MODEL OF THE CASCADE TRANSVERSE THERMOELECTRIC AND GALVANO-THERMOMAGNETIC COOLERS

Cascade transverse cooler consists of the separate rectangular CE (Fig. 4), the sizes of which are chosen in such a way as to achieve the usual cascade cooling while CE are arranged one above the other [4]. In this case the heat extracted on the "hot" face of each element is a thermal load of thermal elements.

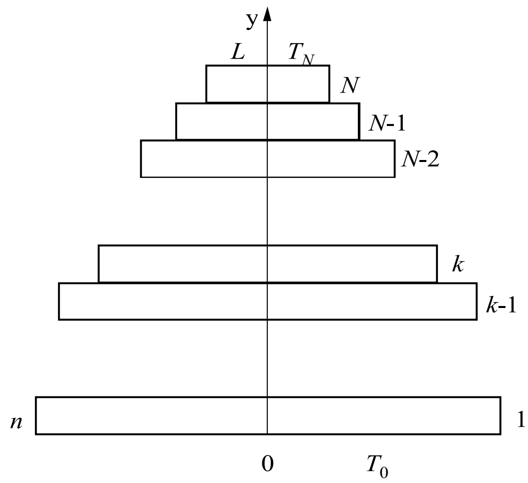

Fig. 4 *– Schematic diagram of the cascade ATC*

Electrical part of the cascade cooler is the following: separate thermal elements of the cooler are not only in the perfect thermal contact, but also in the electric one. Electric contact, as considered in the cited publications, does not influence the potential and temperature distributions and gives the possibility to use the only one power source for all thermal elements. Such idea of cascading leads to that the transverse cooler should have the side faces of a special shape. Experimental investigations show that such a cooler has the right of existence: for example, it gives deeper cooling than the same cooler of the squared shape [7, 8]. But a number of questions arise while analyzing the model of cascade transverse cooler, which give rise to doubts about its validity.

Condition of the constancy of the electric field in cascades does not hold since when constructing such CE the bulk current leads made of materials with high electric conductivity, and so, with high thermal conductivity are required, that will inevitably lead to the thermal interaction with environment. This interaction changes the temperature distribution in CE, i.e., the distribution ceases to be one-dimensional, that, in turn, leads to violation of the condition of the constancy of the electric field, and as a result, to denial of the cascading idea.

Cascading can be more realistic if separate rectangular thermal elements were electrically isolated from each other, but at the same time were in the perfect thermal contact with each other. In this case the condition $j_i = \text{const}$ fulfills better than the condition $E_i = \text{const}$. Moreover, currents in each thermal element are independent. The cascading scheme can be represented as follows. In Fig. 4 we present the middle part (cross-section) of the long enough cooler, for which temperature is one-dimensional, i.e., it depends on the y only. Each thermal element has its own current density j_k. Then under the condition that the kinetic coefficients $\alpha_{12} = \alpha$, $\chi_{22} = \chi$ and $\rho_{11} = \rho$ are constant, the task for ATC is the following:

$$\frac{d^2T_k}{dy^2} + b_k = 0\ , \tag{10}$$

where T_k is the temperature in the k-th thermal element, $b_k = \rho_k j_k^2 / \chi_k$.
Equation (10) has to be solved with the boundary conditions

$$T_1(0) = T_0,\ T_1(N) = T_N, \tag{11}$$

which should be supplemented by the join conditions of the temperatures and heat flows at the interface between CE. Thus, for example, the heat flow and the temperature should be continuous between the $(k-1)$ and k-th thermal elements that is mathematically expressed by the conditions

$$\left(-\chi_{k-1}\frac{dT_{k-1}}{dy} + \alpha_{k-1}T_{k-1}j_{k-1}\right)S_{k-1} = \left(-\chi_k\frac{dT_k}{dy} + \alpha_k T_k j_k\right)S_k\ ,\ T_{k-1} = T_k\ ,$$

where S_k is the base area of the k-th thermal element.
Here should be as many conditions as interfaces. Difference between S_k and S_{k-1} leads to that the side surfaces will have the form, which is different from the plane one.

As seen from the aforesaid, this task is not a simple one, and the more number of separate CE, the more complicated its solution is. To our opinion, the model proposed describes in more detail the real physical situation.

Obviously, the mentioned task should be solved using a computer, setting the areas S_k, material constants and currents. If program is successful, computer calculations can lead to the maximum temperature drop at the fitted optimal currents in separate CE.

Experimental realization of the proposed cascade cooler is not a simple task: it is necessary to provide the reliable thermal leads between separate thermal elements and their independent supply as well.

As known, in practice one can be restricted by a small amount of cascades as it is done for the usual Peltier coolers.

5. CONCLUSIONS

1. From the point of view of one-dimensional temperature model under the condition of the constancy of the electric current in separate AT CE the maximum temperature drop of the two-stage anisotropic cooler is found. Calculation results are generalized for the case of multistage anisotropic cooler.
2. Influence of the strong anisotropy of the thermal conductivity on the temperature field of anisotropic thermal element subject to its sizes and value of the temperature drop is studied.

REFERENCES

1. M.P. Lenyuk, O.G. Danalakiy, *Visnyk Donbas'koi derzhavnoi mashynobudivnoi akademii. Progresyvni tekhnologii* No64, 24 (2009).
2. E.Kamke, *Spravochnik po obyknovennym differentsial'nym uravneniyam (Per. s nem.)* (M.: Nauka: 1971).
3. V.M. Matyega, O.G. Danalakiy, *Visnyk VPI* **3** No3, 63 (2008).
4. L.I. Anatychuk, *Termoelementy i termoelektricheskie ustroistva: Spravochnik* (K.: Nauk. dumka: 1979).
5. E.V. Osipov, *Tverdotel'naya kriogenika* (K.:Nauk. dumka: 1977).
6. V.V. Gutz, *Naukovy visnyk Chernivets'kogo universytetu. Fizyka. Elektronika* No79, 107 (2000).
7. T.C. Harman, *VDI Berichte* **3**, 667 (1983).
8. C.F. Kooi, R.B. Horst, K.F. Cuff, *J. Appl. Phys.* **39** No9, 4257 (1968).