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PROPERTIES OF ANISOTROPIC STRATIFIED STRUCTURE AT THE WAVE PROPAGATION ALONG THE INTERFACE

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Plane-parallel anisotropic stratified structure is considered. Dependence of the reflection coefficient on the incident angle and anisotropic axis orientation is studied. It is proposed to use such structure as a valve and a frequency detector in the optical band.

Keywords: OPTICAL ISOLATOR, OPTICAL FREQUENCY DETECTOR, ANISOTROPIC STRATIFIED STRUCTURE, WAVE "PENETRATION".

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1. INTRODUCTION

Propagation of electromagnetic waves is widely studied, and there are a lot of publications and monographs [1-3] devoted to this question.

Both linear [4-14] and nonlinear [15] problems are solved for the layered structures. Reflection from the arbitrary-shaped objects, calculation and design of the waveguide structures, valves, filters [4-14, 18-27], etc. are related to the linear problems. The nonlinear problems are: generation of the second and the third harmonics, Bloch oscillations, bistable structures [15] and others.

Linear problems are solved both by the numerical and analytical methods. The integral equation method and the FDTD method are the most widespread numerical methods, and the translation matrix method and the small parameter methods are the widely used analytical techniques. The translation matrix method and the transmission and reflection matrix methods [16, 17] used in the present paper are the most popular analytical methods for the prediction of electromagnetic wave behavior in considered mediums. They are 4×4 translation matrix and 2×2 reflection (transmission) matrix for anisotropic crystals.

Among the problems solved in the layered medium theory the transmission [6, 7] and reflection [8, 9, 18-21] effects both of the normally incident wave and of the wave incident at an oblique angle are the most often studied ones. Effects of the total internal reflection at any incident angle [18-21], existence of inhomogeneous waves [15], etc. are found for anisotropic stratified mediums. Study of the surface waves in such mediums [22-25] are of interest as well.

Layered mediums are also widespread in practice. They are used in optics, communication engineering, microwave radio engineering, etc. In particular, they found a wide application in different modern communication systems. Various detectors, microstrip lines, filters, radiator coatings, valves [8-14] are realized on the basis of layered mediums. However, in spite of the great interest in this field, a lot of problems are far from being solved. For example,

currently the frequency detector of the optical band is not proposed; valve operation is based on the reflection effect at an oblique angle that substantially increases their geometric sizes.

In the present paper we have studied the dependence of the reflection coefficient on the incident angle, the inclination angle of anisotropic axis, and on the angle between the incident plane and the plane, which contains the anisotropic axis for one-dimensional anisotropic stratified structure. Wave propagation parallel to the plate in one-dimensional anisotropic stratified structures is also studied, effect of the so-called “penetration” is described, and the frequency dependence of the reflection coefficient at the incident angles $\pm 90^\circ$ is investigated. Based on the studied structures the new valve type and the frequency detector of the optical band are proposed.

2. STATEMENT OF THE PROBLEM

In the present paper we study the anisotropic stratified structure with both electrical and magnetic anisotropies, which is described by the tensors ε and μ in gyrotropic form

$$a = \begin{vmatrix} a_{11} & j \cdot a_{12} & 0 \\ -j \cdot a_{12} & a_{11} & 0 \\ 0 & 0 & a_{33} \end{vmatrix}, \quad (1)$$

where a represents either the magnetic permeability tensor μ or the dielectric permittivity tensor ε .

The problem geometry is shown in Fig. 1, where xyz is the coordinate system connected with the interface normal; $x'y'z'$ is the coordinate system connected with the anisotropy axis; θ is the inclination angle of the anisotropy axis; φ is the angle between the plane involving the anisotropy axis and the incident plane; α_{inc} is the incident angle; k_{inc} , k_{refl} are the wave vectors of the incident and reflected waves, respectively.

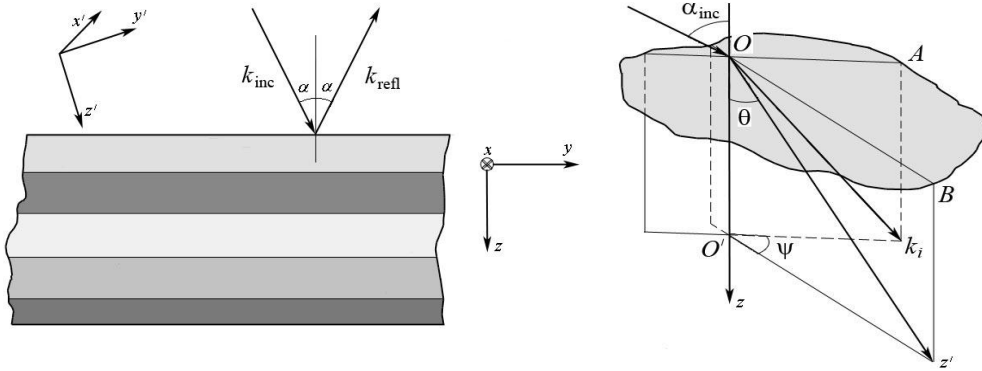


Fig. 1 – The problem geometry

Expression (1) is written in the coordinate system connected with the anisotropy axis z' . As a result of its transformation to the xyz coordinate system connected with the interface normal, the tensor a takes the form

$$a_{norm} = \begin{vmatrix} a_{11} & ja_{12} \cos \theta & -ja_{12} \sin \theta \\ -ja_{12} \cos \theta & a_{33} \sin^2 \theta + a_{11} \cos^2 \theta & (a_{33} - a_{11}) \cos \theta \sin \theta \\ ja_{12} \sin \theta & (a_{33} - a_{11}) \cos \theta \sin \theta & a_{11} \sin^2 \theta + a_{33} \cos^2 \theta \end{vmatrix}. \quad (2)$$

Thus, in this work, in fact, we investigate the medium with all nine non-zero tensors of the dielectric permittivity and magnetic permeability.

For study we use the well-known methods of 4×4 translation matrix [16] and 2×2 reflection matrix [17].

The main problem of this paper is description of new possible applications of the considered structure. Transmission of the plane harmonic wave parallel to the plate interface is mainly studied.

3. NUMERICAL RESULTS

In this section we present the investigation results of the transmission and reflection of the plane harmonic wave at its propagation parallel to the interface. Dependences of the modulus of reflection coefficient on the angles α_{inc} , θ , φ and the frequency are considered.

3.1 Complex wave numbers

First of all, the results of the numerical calculations show that the wave numbers of the wave passed to the plate can be complex or imaginary at any incident angles. This phenomenon is already described in [18-21], but here we present its mathematical justification.

Mathematically the complex wave numbers are the result of the presence of all nine tensor components (2) and of their dependence on the angles θ , α_{inc} and φ . Really, increase in the angle θ leads to a monotonic increase in the imaginary components μ_{13} , μ_{31} of tensor (2) and to a monotonic decrease in the imaginary components μ_{12} , μ_{21} . At the same time the real component μ_{22} monotonically decreases from μ_{xx} to μ_{zz} ; the real component μ_{33} monotonically increases from μ_{zz} to μ_{xx} ; the real components μ_{32} , μ_{23} monotonically increase from $\theta = 0^\circ$ to $\theta = 45^\circ$ and monotonically decrease from $\theta = 45^\circ$ to $\theta = 90^\circ$. Moreover, the angle φ between the incident plane and the plane involving the anisotropy axis is taken into account in the tangential components of the wave vector $k_x = k_{inc} \cos \alpha_{inc} \cos \varphi$ and $k_y = k_{inc} \cos \alpha_{inc} \sin \varphi$. All these factors influence the coefficients of the fourth-order dispersion equation

$$k_z^4 + a_3 k_z^3 + a_2 k_z^2 + a_1 k_z + a_0 = 0, \quad (3)$$

the solution of which is the normal components of the wave vector. Investigation of the real and imaginary roots of equation (3) can be performed analytically, but expressions are found to be so cumbersome that it is more efficiently to use the numerical analysis for certain material parameters.

In this case the numerical study gives the following exotic results:

- wave numbers of both forward waves are complex or imaginary, and they are real for the backward waves;
- wave numbers of both backward waves are complex or imaginary, and they are real for the forward waves;
- wave numbers of one forward and one backward waves are complex, and they are real for two other waves;

- three of four wave numbers can have positive (negative) real parts.

Note, appearance of the imaginary part of the wave number does not imply about the losses in the medium. Really, wave in anisotropic medium is considered as a result of superposition of four own waves, but not as each wave separately. In [27] established that the complex wave numbers indicate the existence of the so-called inhomogeneous (complex) wave in anisotropic materials.

3.2 Total reflection phenomenon

As seen from Fig. 3, the total reflection phenomenon from anisotropic layered plate can be observed at any incident angle of the plane harmonic wave. This effect is not new and is widely described in [18, 19]. We only note, that the regions of total reflection correspond to the complex (imaginary) wave numbers of at least one of four own waves.

3.3 Resonant behavior of the reflection coefficient

In spite of many publications in the field of inhomogeneous anisotropic structures, dependence of the reflection and transmission coefficient on the incident angle and orientation of the anisotropy axis is almost not studied. Little attention to this question was devoted, for example, in [18, 19, 27].

As seen from Fig. 3, in contrast to isotropic mediums dependence of the reflection coefficient on the incident angle is not necessarily the monotonic one, and it can have the resonant behavior. Physical factors, which influence such dependence behavior, are the following:

- the interference phenomenon, if layer thickness is more than the wavelength;
- difference between phase velocities of all forward and backward waves;
- the multiple reflection effect in the layered structure;
- possibility of existence of the complex wave numbers in anisotropic medium.

As a result, dependence of the reflection coefficient on the incident angle can have any behavior. Thus, minimum of the reflection coefficient can be observed at any incident angle.

4. EFFECT OF THE WAVE "PENETRATION" INTO ANISOTROPIC PLATE

Consider the harmonic wave propagation along the layered anisotropic medium (Fig. 2). Calculations show that at $\alpha_{inc} = \pm 90^\circ$ the reflection coefficient is not necessarily equal to 1 (Fig. 3).

That is, the effect of wave "penetration" into anisotropic plate is observed. This phenomenon is not connected with heterogeneity or anisotropy of the medium. Really, even for the case of the linearly polarized incident wave the vectors E_{inc} and H_{inc} have non-zero tangential components (Fig. 2) if these components are directed at an oblique angle to the incident plane (are not perpendicular). Exception is the cases when these vectors coincide with the x - or y -axes (in this case the tangential components either of the electric field or of the magnetic one are absent) and, taking into account the continuity conditions of tangential components of electromagnetic field on the interface, the wave does not penetrate into anisotropic medium.

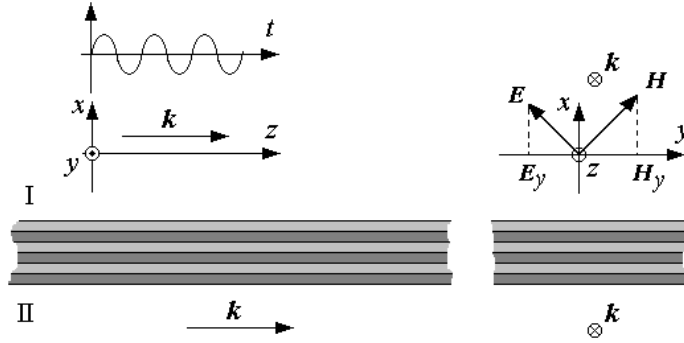


Fig. 2 – Illustration of the structure, which “retracts” the wave

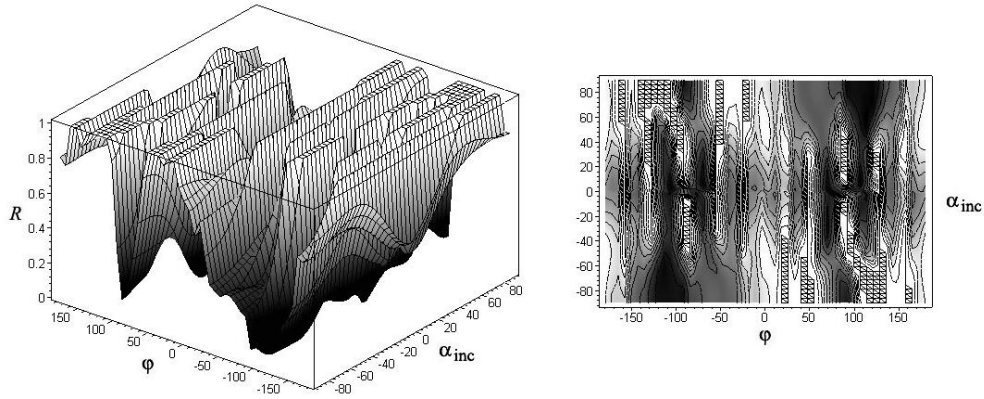


Fig. 3 – Dependence of the reflection coefficient on the incident angle and on the angle between the incident plane and the plane involving the anisotropy axis for the structure, which is formally described as $A2 \times A1 \times 2 \cdot A2 \times 3 \cdot (A1 \times A2) \times A2 \times A1 \times 6 \cdot (A1 \times A2) \times 2 \cdot A2 \times 2 \cdot A1 \times 3 \cdot A2 \times A1 \times A2$; $\theta = 5^\circ$, $f = 54 \cdot 10^{12} \text{ Hz}$, parameters of the first layer (A1): $d_1 = 10 \text{ nm}$, $\varepsilon_{11} = 6,5\varepsilon_0$, $\varepsilon_{12} = 6,2\varepsilon_0$, $\varepsilon_{33} = 0,99\varepsilon_0$, $\mu_{11} = 2,9\mu_0$, $\mu_{12} = 2,7\mu_0$, $\mu_{33} = 0,99\mu_0$; parameters of the second layer (A2): $d_2 = 200 \text{ nm}$, $\varepsilon_{11} = 2,29\varepsilon_0$, $\varepsilon_{12} = 1,15\varepsilon_0$, $\varepsilon_{33} = 0,99\varepsilon_0$, $\mu_{11} = 2,8\mu_0$, $\mu_{12} = 2,6\mu_0$, $\mu_{33} = 0,99\mu_0$

Consider behavior of the wave vector k . Tangential components of k on the interface are not equal to zero as well. Then, taking into account the continuity conditions of tangential components we obtain the dispersion equation for homogeneous anisotropic layer

$$c_4 k^4 + c_2 k^2 + c_0 = 0, \quad (4)$$

where

$$c_4 = \omega^2 [(\varepsilon_{xx}\mu_{zz} + \varepsilon_{zz}\mu_{xx} - \varepsilon_{xx}\mu_{xx} - \varepsilon_{zz}\mu_{zz})(\sin^2 \theta \cos^2 \varphi + \cos^2 \varphi) + (2\varepsilon_{zz}\mu_{zz} - \varepsilon_{xx}\mu_{zz} - \varepsilon_{zz}\mu_{xx})(\cos^2 \theta + 2\sin^2 \theta \cos^2 \varphi) - \varepsilon_{zz}\mu_{zz}];$$

$$\begin{aligned}
c_2 &= \omega^4 \{ [\varepsilon_{xx} \varepsilon_{zz} (\mu_{xx}^2 - \mu_{xy}^2) + \mu_{xx} \mu_{zz} (\varepsilon_{xx}^2 + \varepsilon_{xy}^2)] (1 - \sin^2 \varphi \sin^2 \theta) - \\
&\quad - 2 \varepsilon_{zz} \mu_{zz} (\varepsilon_{xx} \mu_{xx} + \varepsilon_{xy} \mu_{xy}) \sin^2 \varphi \sin^2 \theta \}; \\
c_0 &= \omega^6 \varepsilon_{zz} \mu_{zz} (\varepsilon_{xx}^2 \mu_{xy}^2 + \varepsilon_{xy}^2 \mu_{xx}^2 - \varepsilon_{xx}^2 \mu_{xx}^2 - \varepsilon_{xy}^2 \mu_{xy}^2).
\end{aligned} \tag{5}$$

The given expression is written for the total wave vector in homogeneous anisotropic medium, and not for its z -component. It is obvious from (4), that the wave vector k is not equal to the tangential component $k_\tau = \sqrt{k_x^2 + k_y^2}$. Therefore, both the tangential and the normal components of the wave vector at the plane harmonic wave propagation along the interface in the medium 1 exist in anisotropic medium. Even for a special case, when the anisotropy axis belongs to the plane of the incident wave, we have the wave vectors of two own waves

$$\begin{aligned}
k_{1f} &= \omega \sqrt{\mu_0 (\varepsilon_{xx} - \varepsilon_{xy}^2 / \varepsilon_{xx})}, \\
k_{2f} &= \omega \sqrt{\mu_0 \varepsilon_{zz}}.
\end{aligned} \tag{6}$$

As seen from (6), $k_{2f} < k_0$ always, since $\varepsilon_{zz} < \varepsilon_0$ for all known materials. Taking into account that $k_{2f}^2 = k_x^2 k_y^2 k_z^2$, $k_{z \text{ isotr}} = 0$, $k_{x \text{ isotr}} = k_x$ and $k_{y \text{ isotr}} = k_y$, k_{z2} component should certainly be the imaginary one. If $\varepsilon_{xx} - \varepsilon_{xy}^2 / \varepsilon_{xx} < \varepsilon_0$, k_{z1} is the imaginary component, and wave does not propagate in anisotropic medium, i.e., the reflection coefficient is equal to 1. If $\varepsilon_{xx} - \varepsilon_{xy}^2 / \varepsilon_{xx} = \varepsilon_0$, then $k_{z1} = 0$. In the case when $\varepsilon_{xx} - \varepsilon_{xy}^2 / \varepsilon_{xx} > \varepsilon_0$, the normal component of the transmitted wave is real and, so, the given wave propagates in anisotropic medium, i.e., the reflection coefficient from the plate is less than 1.

In this case the angle between the wave vector and the normal for the first transmitted wave is equal to

$$\alpha_{1 \text{ anisotr}} = \arcsin \frac{k_0}{k_{1f}} = \arcsin \sqrt{\frac{\varepsilon_0}{\varepsilon_{xx} - \varepsilon_{xy}^2 / \varepsilon_{xx}}} = 0. \tag{7}$$

For the second wave with imaginary wave vector the concept of the wave vector does not make sense.

Note, that the same effect at the wave propagation along the plate is possible in isotropic materials as well, since the tangential components of the field and the wave vector on the interface are not equal to zero, and, therefore, the continuity condition of the corresponding components holds.

Now the consider dependence of the reflection coefficient on the incident angle represented in Fig. 4. Since at $\alpha_{inc} = 90^\circ$ the reflection coefficient is equal to 0,06 on the frequency 550 THz, and, therefore, the wave almost completely passes from the medium I to the medium II, and at $\alpha_{inc} = -90^\circ$ the reflection coefficient is equal to 1 and the wave totally reflects from the interface, then the such structure can be used as an optical isolator (Fig. 5). Really, in the medium II the wave will propagate only in the positive direction of the y -axis. In this case the total reflection at $\alpha_{inc} = -90^\circ$ is conditioned by the complex wave number in anisotropic medium.

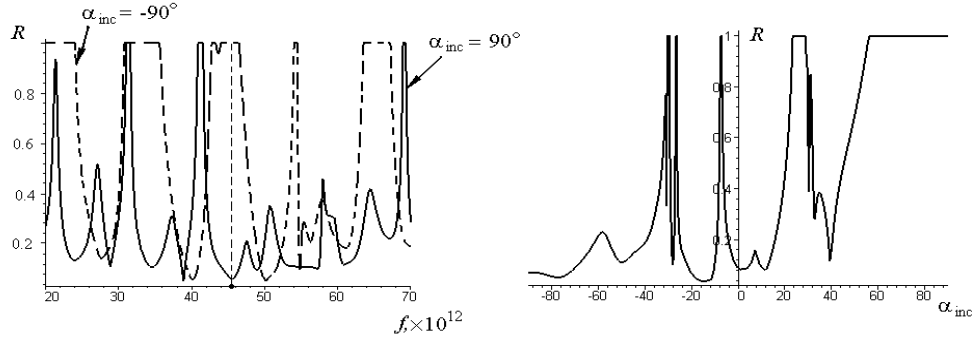


Fig. 4 – Amplitude-angle characteristic of a valve at $f = 54 \cdot 10^{12}$ Hz, $\varphi = 125^\circ$, $\theta = 5^\circ$. Parameters of the first layer (A1): $d_1 = 10$ nm, $\varepsilon_{11} = 6,5\varepsilon_0$, $\varepsilon_{12} = 6,2\varepsilon_0$, $\varepsilon_{33} = 0,99\varepsilon_0$, $\mu_{11} = 2,9\mu_0$, $\mu_{12} = 2,7\mu_0$, $\mu_{33} = 0,99\mu_0$; parameters of the second layer (A2): $d_2 = 200$ nm, $\varepsilon_{11} = 2,29\varepsilon_0$, $\varepsilon_{12} = 1,15\varepsilon_0$, $\varepsilon_{33} = 0,99\varepsilon_0$, $\mu_{11} = 2,8\mu_0$, $\mu_{12} = 2,6\mu_0$, $\mu_{33} = 0,99\mu_0$. The structure is formally described as $A2 \times A1 \times 2 \cdot A2 \times 3 \cdot (A1 \times A2) \times A2 \times A1 \times 6 \cdot (A1 \times A2) \times 2 \cdot A2 \times 2 \cdot A1 \times 3 \cdot A2 \times A1 \times A2$

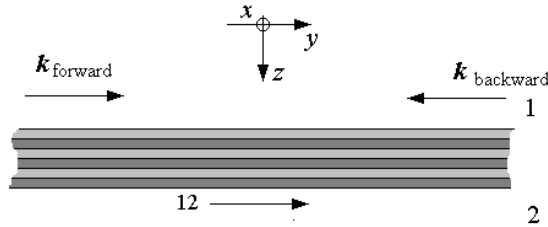


Fig. 5 – Operating principle of the optical isolator

5. OPTICAL ISOLATOR

As noted above, the effect of wave “penetration” is also possible for isotropic plate as well. However, at the wave propagation in the opposite direction the same phenomena will be observed above the anisotropic plate, and the isotropic structure will not exhibit the valve properties.

Nowadays valves of the optical band based on anisotropic structures [6, 7] already exist, but the reflection at some angle (which is not equal to $\pm 90^\circ$) was used there, and as a result the device was found to be sufficiently bulky. Proposed in the present paper use of the valve properties at $\pm 90^\circ$ essentially decreases the device dimensions.

Here we do not consider the questions connected with the polarization of the transmitted wave, and only describe the phenomena of the wave transmission and reflection from anisotropic layered medium with an arbitrary direction of the anisotropy axis. However, it is evident that in the medium II the wave will have the longitudinal components of the electromagnetic field (y -components). These questions are the subjects of further investigation.

Since by selection of the layer thickness and parameters the given effect can be obtained in any waveband, the corresponding device can be designed in the gigahertz, terahertz and in the optical bands.

6. OPTICAL FREQUENCY DETECTOR

Presented in publications the amplitude-frequency characteristics of layered structures have the wave-like behavior [2, 3] and, as a rule, are within the limits from 0 to 1. And the case with $\alpha_{inc} = \pm 90^\circ$ was not considered at all. However, as it was indicated before, this dependence can have any behavior. In this section we study the dependence of the reflection coefficient on the frequency at the plane harmonic wave propagation along anisotropic layered plate. The most interesting from the practical point of view results are presented in Fig. 6.

Amplitude-frequency characteristic of the reflection coefficient on the plane-parallel anisotropic plate with parameters stated in Fig. 6 at $\alpha_{inc} = 90^\circ$ has resonant behavior, and in the resonance region can be considered as the linear one. Here intervals with $R = 1$ are defined by the complex wave numbers, and linearity of the dependence on the interval from $\alpha_{inc} = 58^\circ$ to $\alpha_{inc} = 70^\circ$ is the result of multiple reflection in the layered medium.

This fact allows to use the considered structure as an optical signal detector. Really, if the carrier frequency of the signal is f_0 , then at small frequency deviation Δf the reflection coefficient varies according to the law of signal variation within $\pm \Delta R$ (Fig.6). Therefore the amplitude of reflected signal varies according to the same law. Thus, the considered structure transforms the frequency-modulated signal into the amplitude-modulated one. Then the amplitude-modulated oscillation can be detected by known methods [8, 9].

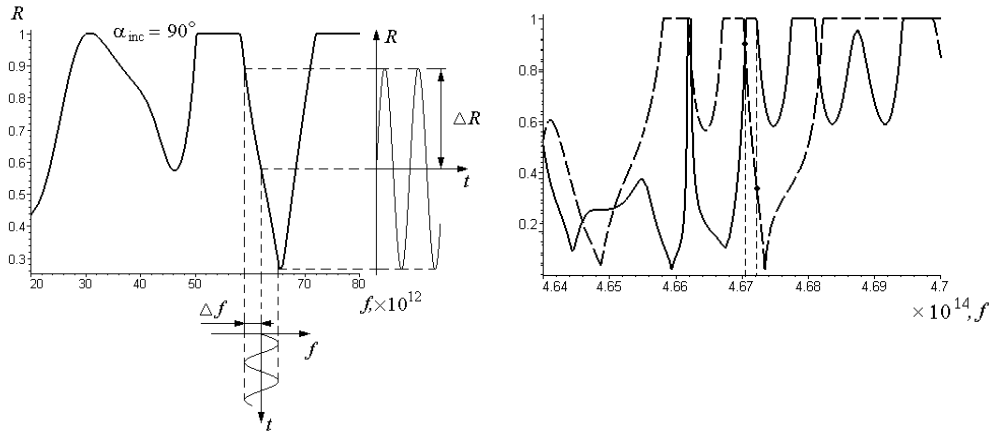


Fig. 6 – Illustration of the operating principle of the optical band frequency detector; considered plate consists of 12 two-layer periods at $\theta = 5^\circ$; $\varphi = 87^\circ$; parameters of the first layer: $d = 100$ nm; $\varepsilon_{11} = 2,5$; $\varepsilon_{12} = 2,2$; $\varepsilon_{33} = 0,99$; $\mu_{11} = 1,9$; $\mu_{12} = 1,7$; $\mu_{33} = 0,99$; parameters of the second layer: $d = 50$ nm; $\varepsilon_{11} = 2,29$; $\varepsilon_{12} = 2,15$; $\varepsilon_{33} = 0,99$; $\mu_{11} = 2,8$; $\mu_{12} = 2,6$; $\mu_{33} = 0,99$

Considered principle can find the practical application while transmitting analog signals in the terahertz band. Here one should take into account that the reflected wave will not be the linearly polarized one any more.

In Fig. 6a we present dependence in the terahertz band, and in Fig. 6b – in the optical band. Thus, selecting the structure parameters the frequency-modulated signal detection can be done in any waveband.

It is important to note, that nowadays in optical communication systems the amplitude modulation only is used. Proposed principle allows to use the frequency modulation in communication systems of the terahertz band.

7. CONCLUSIONS

In present paper, we consider the “penetration” effect of the plane harmonic wave propagating parallel to anisotropic layered plate, describe the valve properties of such structure and possibility of their practical application in the optical communication systems. Investigation of the amplitude-frequency characteristic of the structure allowed to conclude about the possibility of its application as the frequency detectors of the terahertz and optical bands.

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