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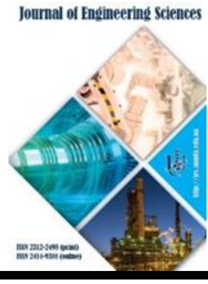

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Effect of the Microorganisms Dynamics on the Base Subsidence of the Solid Household Waste Storage During Consolidation

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Abstract. The effect of the dynamics of the development of the microorganism biomass on the subsidence of the surface of the mass of the porous medium at the base of the storage of solid household waste was studied using mathematical and computer modeling methods. The repository of solid household waste is considered a source of the spread of organic pollutants that contribute to the development of the biomass of microorganisms. The subsidence model is considered from the theory of filtration consolidation of porous media. For this purpose, a corresponding boundary value problem in the domain with a free-moving boundary is formed. The classical equation of filtration consolidation is modified for the case of variable porosity due to biomass change. The finite element method allowed for solving the resulting mathematical model numerically in the form of a boundary value problem for the system of parabolic equations in the variable domain. Based on the developed algorithms, a software package for numerical experiments was created where the effect of microorganisms on the subsidence dynamics of a porous medium was investigated. Numerical experiments on a model example showed that the presence of microorganisms in soil pores slows the dissipation of excess pressure. However, this does not lead to a slowdown in subsidence but, on the contrary, to a particular increase of up to 18 % compared to the case of neglecting bioprocesses. From the point of view of the physics of the processes, this is explained by the increase in biomass, which leads to an increase in pressure, thus increasing the volume of filtered pore fluid and, hence, increasing subsidence.

Keywords: clean water and sanitation, biomass, finite element method, kinematic boundary condition.

1 Introduction

Repositories of solid household waste are an integral element and consequence of human activity. As noted in [1], “Municipal solid waste landfills (MSWLs) are a critical component of the engineered environment, serving as a terminal repository for commercial, consumer, and industrial solid wastes”.

The functioning and operation of waste storage create risks for surrounding environments, which can be evaluated quantitatively. The impact of urban solid waste landfills on the environment using the weighted environmental index (WEI) was analyzed in [2].

The results of the research of the Valencia Region (Spain) showed that the ecological value of the territory decreased, that municipal waste landfills have a negative effect on the environment, and that the use of WEI enabled the authors to evaluate quantitatively the level of such impact.

One of the risk factors is that solid municipal (household) waste storages are potential sources of organic pollution of the soil base on which they are built and then of groundwaters.

Aggressive organic substances such as acids [3] can increase the hydraulic conductivity of clay geobarriers and, as a result, the increased outflow of pollutants beyond the boundaries of waste storage facilities.

2 Literature Review

Organic pollution also creates favorable conditions for developing microorganisms in porous media [4]. Intense development of microorganisms leads to bioclogging effects in the porous environment [5], including pharmaceutically active substances [6].

The review [7] noted that the decrease in the permeability of porous media due to bioclogging is one of the features of biomass development in these media. The laboratory experiments [8] show that bacterial biomass due to bioclogging can affect the hydraulic properties of saturated porous media. Microbes can cause both biocementation and bioclogging effects in geomaterials. It was shown in [9] that this depends on the biochemical conditions of the porous media, with biocementation affecting the strength, and bioclogging, which depends on the quantity of biomaterial, mainly results in the decrease of hydraulic conductivity of the porous medium. The possibility of using microorganisms to control biofilms and calcite formation to reduce the hydraulic conductivity of geobarriers in the bases of waste storages was shown in [10].

Reviews on the effect of bioclogging on the hydraulic conductivity of porous media are presented in [11–13].

A significant weight of waste leads to the development of compaction processes (consolidation) of both soil mass at the foundation of waste storage facilities and the waste mass itself as a porous media. Much research, including field experiments, was devoted to the problems of compaction of porous media in general and soils in particular [14].

In recent years, mathematical models of consolidation have increasingly considered the specifics of physicochemical conditions and the state of the porous medium, making the models vary from the classical ones. One-dimensional nonlinear consolidation of two-layer soil was investigated in [15] based on assumptions about the nonlinear behavior of soil characteristics. The problem of soft soil consolidation with a drained lower boundary at various values of its permeability degree was considered in [16]. A one-dimensional model of consolidation of soft soil under conditions of collective horizontal drainage and application of non-stationary vacuum pressure was also proposed in [17]. It was experimentally shown in [18] that the consolidation characteristics of soft clay show significant differences under multi-stage and single-stage loading.

Moreover, although the resulting deformation between multi-stage and single-stage loads is about the same, the consolidation rate for a single-stage load is several times higher than that for a multi-stage load. Considering the impact of the soil's weight on the consolidation process is a feature of [19] reflected in an original mathematical model. A new mathematical model of filtration consolidation based on a fractional-fractal approach was developed in [20].

A peculiarity of the problem is considering the effect of the transfer of inorganic chemicals. It was noted in [21] that the weight of solid waste or its artificial compaction can accelerate the migration of pollutants from landfills to the outside environment.

The physics of the consolidation process in real conditions generally predicts the subsidence of the entire porous medium, including its surface, in the compaction process. Using the unmanned aerial vehicles, it is possible to identify and quantify the subsidence through field experiments and monitoring data, e.g., as shown in [22]. A model that allows simulating the movement of the landfill body based on monitoring data sets from the Global Navigation Satellite System (GNSS) was proposed in [23]. A variational autoencoder was used to predict embankment settlement and pore water pressure based on monitoring data, which was investigated in [24].

An alternative way to determine subsidence is to develop appropriate mathematical models for predictive numerical experiments. However, not all mathematical consolidation models consider the subsidence effect. First of all, the difficulties of practical implementation are related to the change of the consolidation area over time and the need to consider the mathematical model the so-called kinematic boundary condition, which describes the change in the area shape [25].

Similar conditions are also used in mathematical models of many other natural processes, e.g., filtration problems through soil dams or moisture transfer in case of moving soil wetting front.

Mathematical models of the effect of bioclogging on pressure drops in geobarriers were constructed and investigated in [11–13]. Geobarriers were considered from the viewpoint of the elements of engineering solutions for protection from the spread of organic pollutants from household waste storages into porous soil environment as the foundation of these waste storages. However, consolidation processes were not investigated. A mathematical model of the consolidation of soil mass in bioclogging conditions was constructed in [12], which disregards subsidence.

Therefore, the purpose of this article is to study the effect of the dynamics of microorganism development in a porous medium on the distribution of excess pressure and on the subsidence of the surface of the mass of the porous medium.

3 Research Methodology

3.1 A mathematical model of the saturated porous medium compaction considering the effect of the microorganism biomass

The mathematical model will include the filtration consolidation equation proposed in [12] supplemented by a kinematic boundary condition at the moving upper boundary $x = l(t)$ of the soil mass that is being compacted [25]. This results in the following boundary value problem:

$$\gamma a \frac{\partial h}{\partial t} + \frac{\partial e}{\partial B}\frac{\partial B}{\partial t} = (1+e)\frac{\partial}{\partial x}\left[k(h,B)\frac{\partial h}{\partial x}\right],$$

$$x \in \Omega = (0; l),\ t > 0; \quad (1)$$

$$h(x,t)|_{x=l(t)} = \bar{h}_l(t), t \geq 0; \quad (2)$$

$$u(x,t)|_{x=0} = \left(-k(h,B)\frac{\partial h}{\partial x}\right)\Big|_{x=0} = 0, t \geq 0; \quad (3)$$

$$h(x,0) = h_0(x), x \in \bar{\Omega} = [0; l]; \quad (4)$$

$$n\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}\left(nD\frac{\partial c}{\partial x}\right) - u\frac{\partial c}{\partial x} - f_c(c,B) + F_c(x,t);$$

$$x \in \Omega = (0; l),\ t > 0; \quad (5)$$

$$c(x,t)|_{x=l(t)} = \bar{c}_l(t),\ t \geq 0; \quad (6)$$

$$q_c(x,t)|_{x=0} = \left(-nD\frac{\partial c}{\partial x}\right)\Big|_{x=0} = 0,\ t \geq 0; \quad (7)$$

$$c(x,0) = c_0(x), x \in \bar{\Omega} = [0; l]; \quad (8)$$

$$n\frac{\partial B}{\partial t}=\frac{\partial}{\partial x}\left(nD_B\frac{\partial B}{\partial x}\right)-u\frac{\partial B}{\partial x}+f_B(n,c,B);$$
$$x\in\Omega=(0;l),\ t>0; \qquad (9)$$
$$B(x,t)|_{x=l(t)}=\bar{B}_l(t), t\geq 0; \qquad (10)$$
$$q_B(x,t)|_{x=0}=\left(-nD_B\frac{\partial B}{\partial x}\right)\Big|_{x=0}=0,\ t\geq 0; \qquad (11)$$
$$B(x,0)=B_0(x),\ x\in\bar{\Omega}=[0;l]; \qquad (12)$$
$$\frac{dl(t)}{dt}=-\int_0^{l(t)}\frac{\partial u}{\partial x}dz\,; \qquad (13)$$
$$l(t)|_{t=0}=l_0>0, \qquad (14)$$

where h – excess pressure; c – the concentration of the pore solution of an organic chemical substance; B – the concentration of microorganism biomass per unit volume of pore fluid; k – the filtration coefficient of the porous medium, which depends on the pressure and biomass concentration; n, $e=\frac{n}{1-n}$ – the porosity and the void ratio of the medium, respectively; q_c, q_B – flows of organic chemicals and biomass; D – the diffusion coefficient of organic matter; $h_0(t)$, $\bar{h}_l(t)$, $c_0(t)$, $\bar{c}_l(t)$, $B_0(x)$, and $\bar{B}_l(t)$ – known functions; $l_0>0$ – the given initial thickness of the mass of the porous medium; a – the compressibility coefficient of the porous medium; γ – the specific gravity of the liquid; u – the filtration rate, determined by Darcy's law:

$$u=-k(h,B)\frac{\partial h}{\partial x},$$

where D_B – the hydrodynamic dispersion coefficient for the component microorganisms in the water phase; $f_c(c,B)$, $f_B(n,c,B)$ – ratios that determine the kinetics of organic substances and the biomass of microorganisms (e.g., growth and death); $f_c(c,B)$ – the function of the sources of organic substances in the porous medium.

The dynamics of microorganisms in a porous medium in the above mathematical model are described by equation (9). This is a diffusion-type equation, and the justification for its use concerning the problems of microorganism development is presented in [26–28].

Condition (13) is a kinematic boundary condition that describes the change in the position of the moving upper boundary (Figure 1) of the porous medium layer that is being compacted. The initial thickness of the porous medium is given by condition (14).

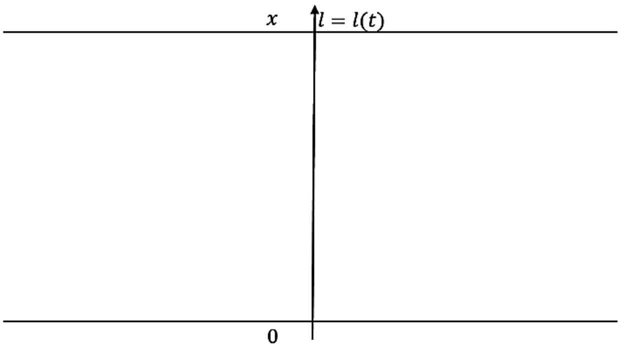

Figure 1 – Layer of a porous medium of variable thickness l

3.2 Finite element method for a problem in a domain with moving boundary

Similarly to [29], let's introduce the following notation: $Q_{\mathcal{T}}=\Omega\times(0;T]$.

Definition 1. The classical solution of the initial boundary value problem (1)–(14) is the triplet of functions $h(x,t)\in\Psi_h$, $c(x,t)\in\Psi_c$, $B(x,t)\in\Psi_B$, which satisfy $\forall(x,t)\in\bar{Q}_{\mathcal{T}}$ equations (1), (5), and (9), as well as initial conditions (4), (8), and (12), respectively.

In the above definition Ψ_h, Ψ_c, and Ψ_B are elements of a set of functions $\psi_h(x,t)$, $\psi_c(x,t)$, and $\psi_B(x,t)$, which together with $\frac{\partial(\cdot)}{\partial x}$ are continuous on closure $\bar{Q}_{\mathcal{T}}$; they have bounded continuous partial derivatives $\frac{\partial(\cdot)}{\partial t}$, $\frac{\partial^2(\cdot)}{\partial x^2}$ on $Q_{\mathcal{T}}$ and satisfy conditions (2), (3), conditions (6), (7), and conditions (10), (11), respectively.

A similar problem for a system of two quasi-linear parabolic equations was investigated in [30] within the framework of the qualitative theory of mathematical physics boundary value problems in regions with moving boundaries. Particularly, the existence and uniqueness of a local time solution in the Hölder space have been proved. The presence in problem (1)–(14) of a system of three equations does not reduce the generality of conclusions similar to [30].

Let H_0 be a space of vector functions $(s_1(x);\ s_2(x);s_3(x))$ each of which components on the interval $(0;l)$ belong to the Sobolev space $W_2^1(\Omega)$, and they acquire zero values at the ends of the segment $[0;l]$ where boundary conditions of the first kind are set for the functions $h(x,t)$, $c(x,t)$ and $B(x,t)$, respectively. Let H be the space of functions $(v_1(x,t);\ v_2(x,t);v_3(x,t))$, each of which components is square integrable together with its first derivatives $\frac{\partial v_i}{\partial t}$, $\frac{\partial v_i}{\partial x}$, $i=\overline{1,3}$ on the interval $(0;l)$, $\forall t\in(0;T]$, $T>0$; they satisfy the same boundary conditions of the first kind as the functions $h(x,t)$, $c(x,t)$ and $B(x,t)$, respectively.

After taking $(s_1(x);\ s_2(x);s_3(x))\in H_0$, multiplying equation (1) and initial condition (4) by $s_1(x)$, integrating them over the segment $[0;l]$, and considering condition (3), it can be obtained:

$$\int_0^l\frac{\gamma a}{1+e}\frac{\partial h}{\partial t}s_1(x)dx+\int_0^l\frac{1}{1+e}\frac{\partial e}{\partial B}\frac{\partial B}{\partial t}s_1(x)dx+$$
$$+\int_0^l k(h,B)\frac{\partial h}{\partial x}\frac{ds_1}{dx}dx=0; \qquad (15)$$

$$\int_0^l h(x,0)s_1(x)dx=\int_0^l h_0(x)s_1(x)dx. \qquad (16)$$

Similarly, for the concentration of organic chemicals and biomass:

$$\int_0^l n\frac{\partial c}{\partial t}s_2(x)dx+\int_0^l nD\frac{\partial c}{\partial x}\frac{ds_2}{dx}dx+\int_0^l u\frac{\partial c}{\partial x}s_2(x)dx=$$
$$=\int_0^l(-f_c(c,B)+F_c(x,t))s_2(x)dx\,; \qquad (17)$$
$$\int_0^l c(x,0)s_2(x)dx=\int_0^l c_0(x)s_2(x)dx\,; \qquad (18)$$

$$\int_0^l n\frac{\partial B}{\partial t}s_3(x)dx + \int_0^l nD_B\frac{\partial B}{\partial x}\frac{ds_3}{dx}dx + \int_0^l u\frac{\partial B}{\partial x}s_3(x)dx = = \int_0^l f_B(n,c,B)s_3(x)dx\,; \quad (19)$$

$$\int_0^l B(x,0)s_3(x)dx = \int_0^l B_0(x)s_3(x)dx. \quad (20)$$

Definition 2. A function $(h(x,t);\ c(x,t); B(x,t)) \in H$ that for any $(s_1(x);\ s_2(x); s_3(x)) \in H_0$ satisfies the integral relations (15)–(20) is called the generalized solution of the boundary value problem (1)–(14).

Let's seek an approximate generalized solution of the boundary value problem (1)–(14) in the form:

$$h(x,t) = \sum_{i=1}^{N} h_i(t)\varphi_{i1}(x)\,, c(x,t) = \sum_{i=1}^{N} c_i(t)\varphi_{i2}(x)\,;$$
$$B(x,t) = \sum_{i=1}^{N} B_i(t)\varphi_{i3}(x), \quad (21)$$

where $h_i(t)$, $c_i(t)$, $B_i(t)$, $i = \overline{1,N}$ – unknown coefficients that depend only on time.

Similarly to [13, 31], if consider (21) as an approximate finite element solution, the functions $\varphi_{i1}(x)$, $\varphi_{i2}(x)$, and $\varphi_{i3}(x)$ for $i = \overline{1,N}$ are polynomial basis functions with a finite carrier.

To simplify further explanations and accordingly for numerical experiments, we assume that the same grid of finite elements is used for the approximate search for $h(x,t), c(x,t), B(x,t)$.

Simultaneously, $\varphi_{i1}(x) = \varphi_{i2}(x) = \varphi_{i3}(x)$, and to avoid double indexing, we denote the indicated basis functions as $\varphi_i(x)$, $i = \overline{1,N}$.

Then, from the weak formulation (15)–(20) of problems (1)–(14), considering the above and (21), it can be obtained:

$$\mathbf{M_1}\frac{d\mathbf{H}}{dt} + \mathbf{M_{12}}\frac{d\mathbf{B}}{dt} + \mathbf{L_1}(\mathbf{B})\mathbf{H}(\boldsymbol{t}) = \mathbf{0}; \quad (22)$$

$$\widetilde{\mathbf{M}}_\mathbf{1}\mathbf{H}^{(\mathbf{0})} = \tilde{\mathbf{F}}_\mathbf{1}; \quad (23)$$

$$\mathbf{M_2}\frac{d\mathbf{C}}{dt} + \mathbf{L_2}(\mathbf{H},\mathbf{B})\mathbf{C}(\boldsymbol{t}) = \mathbf{F_2}(\mathbf{C},\mathbf{B}); \quad (24)$$

$$\widetilde{\mathbf{M}}_\mathbf{2}\mathbf{C}^{(\mathbf{0})} = \tilde{\mathbf{F}}_\mathbf{2}; \quad (25)$$

$$\mathbf{M_3}\frac{d\mathbf{B}}{dt} + \mathbf{L_3}(\mathbf{C},\mathbf{B})\mathbf{B}(\boldsymbol{t}) = \mathbf{F_3}(\mathbf{C},\mathbf{B}); \quad (26)$$

$$\widetilde{\mathbf{M}}_\mathbf{3}\mathbf{B}^{(\mathbf{0})} = \tilde{\mathbf{F}}_\mathbf{3}, \quad (27)$$

where

$$\tilde{\mathbf{F}}_k = \left(\tilde{f}_i^{(k)}\right)_{i=1}^N, \widetilde{\mathbf{M}}_k = \left(\widetilde{m}_{ij}^{(k)}\right)_{i,j=1}^N, \widetilde{m}_{ij}^{(k)} = \int_0^l \varphi_i\varphi_j dx\,;$$

$$\mathbf{M}_k = \left(m_{ij}^{(k)}\right)_{i,j=1}^N, \mathbf{L}_k = \left(l_{ij}^{(k)}\right)_{i,j=1}^N, \mathbf{F}_k = \left(f_i^{(k)}\right)_{i=1}^N;$$

$$k = \overline{1,3}; \mathbf{M}_{12} = \left(m_{ij}^{(12)}\right)_{i,j=1}^N;$$

$$\tilde{f}_i^{(1)} = \int_0^0 h_0\,\varphi_i dx, \tilde{f}_i^{(2)} = \int_0^0 c_0\,\varphi_i dx;$$

$$\tilde{f}_i^{(3)} = \int_0^0 B_0\,\varphi_i dx;$$

$$\mathbf{H} = \left(h_i(t)\right)_{i=1}^N, \mathbf{C} = \left(c_i(t)\right)_{i=1}^N, \mathbf{B} = \left(b_i(t)\right)_{i=1}^N;$$

$$\mathbf{H^{(0)}} = \left(h_i(0)\right)_{i=1}^N, \mathbf{C^{(0)}} = \left(c_i(0)\right)_{i=1}^N;$$

$$\mathbf{B^{(0)}} = \left(b_i(0)\right)_{i=1}^N;$$

$$m_{ij}^{(1)} = \int_0^l \frac{\gamma a}{1+e}\varphi_i\varphi_j dx, m_{ij}^{(12)} = \int_0^l \frac{1}{1+e}\frac{\partial e}{\partial B}\varphi_i\varphi_j dx\,;$$

$$l_{ij}^{(1)} = \int_0^l k(h,B)\frac{d\varphi_i}{dx}\frac{d\varphi_j}{dx}dx, m_{ij}^{(2)} = \int_0^l n\varphi_i\varphi_j dx\,;$$

$$l_{ij}^{(2)} = \int_0^l nD\frac{d\varphi_i}{dx}\frac{d\varphi_j}{dx}dx + \int_0^l u\frac{d\varphi_j}{dx}\varphi_i dx, f_i^{(2)} =$$

$$= \int_0^l (-f_c(c,B) + F_c(x,t))s_2(x)dx\,;$$

$$m_{ij}^{(3)} = \int_0^l n\varphi_i\varphi_j dx, f_i^{(3)} = \int_0^l f_B(n,c,B)\varphi_i(x)dx\,;$$

$$l_{ij}^{(3)} = \int_0^l nD_B\frac{d\varphi_i}{dx}\frac{d\varphi_j}{dx}dx + \int_0^l u\frac{d\varphi_j}{dx}\varphi_i(x)dx.$$

Equations (22)–(27) form a Cauchy problem for a system of nonlinear differential equations of the first order. Finding its solution also requires the use of appropriate discretization schemes. For the simplicity of practical implementation, a fully implicit linearized difference scheme was proven suitable [11, 25]. For example, for the system (24) it has the form:

$$\mathbf{M}_2\left(\mathbf{B}^{(j)}\right)\frac{\mathbf{C}^{(j+1)}-\mathbf{C}^{(j)}}{\tau} + \mathbf{L}_2\left(\mathbf{H}^{(j)},\mathbf{B}^{(j)}\right)\cdot\mathbf{C}^{(j+1)} =$$
$$= \mathbf{F}_2\left(\mathbf{C}^{(j)},\mathbf{B}^{(j)}\right), j = 0,1,2,\ldots,m_\tau - 1.$$

where time segment $[0,T]$ – split into m_τ equal parts with a step $\tau = \frac{T}{m_\tau}$; $\mathbf{H}^{(j)}$, $\mathbf{C}^{(j)}$, and $\mathbf{B}^{(j)}$ – the approximate solution of the Cauchy problem (22)-(27) at $t = j\tau$.

The Crank–Nicolson method may also be applied. However, the practical implementation of this requires iterations. Instead of the Crank-Nicolson method, the predictor-corrector scheme may be used.

3.3 Approximation of the kinematic boundary condition

Condition (13) is not directly convenient for the numerical calculation of subsidence of the soil surface. Therefore, similar to [29], the filtration rate determined according to the law of filtration

$$u = -k(h,B)\frac{\partial h}{\partial x}$$

can be substituted in (13). As a result,

$$\frac{dl(t)}{dt} = \int_0^{l(t)} \frac{\partial}{\partial x}\left[k(h,B)\frac{\partial h}{\partial x}\right]dz.$$

Next, using equation (1), from the above equality, it can also be obtained:

$$\frac{dl(t)}{dt} = \int_0^{l(t)}\left(\frac{\gamma a}{1+e}\frac{\partial h}{\partial t} + \frac{1}{1+e}\frac{\partial e}{\partial B}\frac{dB}{dt}\right)dz. \quad (28)$$

After applying time discretization to (28) in the implicit difference scheme, it can be obtained the following:

$$\frac{l^{(j+1)} - l^{(j)}}{\tau} =$$
$$= \int_0^{l^{(j)}} \left(\frac{\gamma a}{1 + e^{(j+1)}} \frac{h^{(j+1)} - h^{(j)}}{\tau} + \frac{1}{1 + e^{(j+1)}} \left(\frac{\partial e}{\partial B}\right)^{(j+1)} \frac{B^{(j+1)} - B^{(j)}}{\tau} \right) dz,$$
$$j = 0, 1, 2, \ldots$$

Then,

$$l^{(j+1)} = l^{(j)} + \int_0^{l^{(j)}} \left\{ \frac{\gamma a}{1 + e^{(j+1)}} \left(h^{(j+1)} - h^{(j)}\right) + \frac{1}{1 + e^{(j+1)}} \left(\frac{\partial e}{\partial B}\right)^{(j+1)} \left[B^{(j+1)} - B^{(j)}\right] \right\} dz,$$
$$j = 0, 1, 2, \ldots \quad (29)$$

where $(j+1)$ is searched for t, using the position of the upper limit $l^{(j)}$.

The integral in (29) can be found using numerical integration formulas. Concretization $e^{(j+1)}$ and $\left(\frac{\partial e}{\partial B}\right)^{(j+1)}$ is given in the following points.

Here $l^{(j+1)} = l(t_{j+1})$, $j = 0, 1, 2, \ldots$ The position $l^{(j)}$ in (29) is known, as are the values $h^{(j+1)}$, $e^{(j+1)}$, $B^{(j+1)}$ and $h^{(j)}$, $B^{(j)}$.

According to the proposed algorithm, the values of pressure and biomass concentration on the time layer $(j + 1)$ are sought using the position of the upper limit $l^{(j)}$. The integral in (29) can be found using numerical integration formulas. The specifics of the calculation of $e^{(j+1)}$ and $\left(\frac{\partial e}{\partial B}\right)^{(j+1)}$ is presented in the following section.

4 Results

The void ratio in a consolidating porous medium depends on the excess pressures. However, in the context of our task, it will also depend on the volume of microorganism biomass. That is $e = e(h, B)$. Then,

$$\frac{de}{dt} = \frac{\partial e}{\partial h}\frac{\partial h}{\partial t} + \frac{\partial e}{\partial B}\frac{\partial B}{\partial t}.$$

According to the principle of effective stress, which is used in the theory of filtration consolidation of porous media, for the one-dimensional case, it can be written:

$$\frac{\partial e}{\partial h} = a\gamma,$$

where a – the compressibility coefficient of the porous medium; γ – the specific gravity of the pore liquid.

Then, using time discretization, it can be obtained:

$$\frac{e^{(j+1)} - e^{(j)}}{\tau} = a\gamma \frac{h^{(j+1)} - h^{(j)}}{\tau} + \left(\frac{\partial e}{\partial B}\right)^{(j+1)} \frac{B^{(j+1)} - B^{(j)}}{\tau},$$
$$j = 0, 1, 2, \ldots, m_\tau - 1,$$

or

$$e^{(j+1)} = a\gamma\left(h^{(j+1)} - h^{(j)}\right) + \left(\frac{\partial e}{\partial B}\right)^{(j+1)} \left(B^{(j+1)} - B^{(j)}\right) + e^{(j)},$$
$$j = 0, 1, 2, \ldots, m_\tau - 1.$$

For the dependence of the filtration coefficient on the void ratio, which is indirectly expressed in the dependence $k = k(e) = k(h, B)$, we use the Kozeny–Carman equation [25] is as follows:

$$k^{(j+1)} = k_0 \frac{1 + e_0}{1 + e^{(j+1)}} \left[\frac{e^{(j+1)}}{e_0}\right]^3,$$

where e_0 – the initial value of the void ratio.

To convert the biomass concentration to volume, it can be assumed that the biovolume is 80 % water and the rest 20 % of the dry mass is 50 % carbon (bio-carbon) [30]. Then,

$$n_B = \frac{B}{0.8\rho_w + 0.2 \cdot {\rho_c}/{2}},$$

where n_B – the biomass volume per unit volume of the porous medium; ρ_w – the density of water; ρ_c – the density of bio-carbon.

In the numerical experiments ρ_c = 100 kg/m^3 [31]. After considering the dependence

$$\frac{\partial e}{\partial B} = \frac{\partial e}{\partial n} \cdot \frac{\partial n}{\partial B} = -\frac{1}{(1-n)^2} \frac{1}{0.8\rho_n + 0.2\frac{\rho_c}{2}}$$

for $n = n_0 - n_B$ and using the dependence $n = \frac{e}{1+e}$, it can be obtained:

$$\frac{\partial e}{\partial B} = -(1+e)^2 \frac{1}{0.8\rho_n + 0.2\frac{\rho_c}{2}}.$$

According to [32],

$$f_c(c, B) = \frac{k_1 c B}{k_2 + c};\ f_B(n, B, c) = k_3(f_c(c, B) - k_4 B).$$

where the following parameters are introduced:

$$k_1 = B_{max}\left(\frac{\mu_m}{Y_{XS}} + m_s\right);\ k_2 = K_S;$$
$$k_3 = \frac{Y_{XS}}{B_{max}};\ k_4 = m_s B_{max}.$$

In the numerical experiments [32], the following values were used: the specific growth rate μ_m = 1.5·10^{-5} s^{-1}; the maintenance coefficient m_s = 3.0·10^{-5} s^{-1}; the maximum value of biomass B_{max} = 100 kg/m^3; the dimensionless substrate growth yield factor Y_{XS} = 0.045; the Monod saturation constant K_S = 3.5·10^{-5} kg/m^3.

According to [28],

$$D_B = \alpha|u| + \tau_B D_B^m,$$

where α = 0.01 m – the transversal dispersivity of the component in water; $\tau_B < 1$ – the tortuosity (in the considered case, τ_B = 0.5); D_B^m = 1.296·10^{-4} m^2/day – the molecular diffusion coefficient of the component in water.

The boundary and initial conditions for biomass are $\bar{B}_0(t) = B_{max}$, $B_0(x)$ = 1 kg/m^3, and B_{max} = 50 kg/m^3.

The initial filtration coefficient of the porous medium was set k_0 = 2.88·10^{-3} m/day, and the initial porosity n_0 = 0.038. The problem considers a layer of porous medium of a thickness l = 10 m. The step of the variable x was 0.01 m, and the time step τ = 1 day.

Also, in equation (1), a = 5.12·10^{-7} m^2/N – the compressibility coefficient of the porous medium; γ_c = 1·10^4 N/m^2 – the specific gravity of the pore solution.

Initial pressure distribution $h_0(x)$ = 0 m. The unobstructed outflow of pore fluid is ensured at the lower boundary, and a boundary condition of symmetry was set at the upper boundary.

A glucose solution was considered as an organic solution. Initial distribution of glucose concentration in pore water $c_0(x)$ = 0.1 kg/m^3. In the boundary condition (6), on the soil surface $\overline{c}_0(t)$ = 10 kg/m^3. The diffusion coefficient of glucose D = 5.184·10^{-5} m^2/day.

The function of the sources of organic substances in equation (5) was set as follows:

$$F_c(x,t) = 70 + 10(x-1), \frac{kg}{m^3};\ x \in [i; i+0.2];\ i = \overline{1,9}.$$

The porous medium in a state of compaction was considered the previously buried solid household waste that is still a source of organic matter entering the liquid phase due to biodegradation.

In the proposed form, the assignment of the function $F_c(x,t)$ is a model. The pressure was automatically increased by 5 m at t = 50 days, t = 100 days, t = 150 days, and t = 200 days due to the applied load from freshly added waste.

Piecewise quadratic functions were used as the basis functions of the finite element method. The results of numerical experiments are presented in Table 1 and Figures 2–3, which illustrate the general picture of changes in excess pressure and biomass distribution.

The distribution of the concentration of organic substances (in the considered case, glucose), regardless of the presence of the source function $F_c(x,t)$, varies around its minimum values since time t = 50 days, i.e., actual distribution of biomass (Figure 2) and distribution of the concentration of nutritious substances reaches equilibrium after a certain time.

Table 1 – Maximum values of pressure and subsidence for the upper limit

Time t, days	Disregarding bioprocesses		Considering bioprocesses	
	h_m, m	Subsi-dence, cm	h_m, m	Subsi-dence, cm
70	4.13	7.50	6.03	8.73
100	7.20	11.62	8.70	14.46
200	8.13	41.49	8.52	47.89
300	1.13	61.41	2.21	70.36
400	0.21	63.26	0.75	74.60

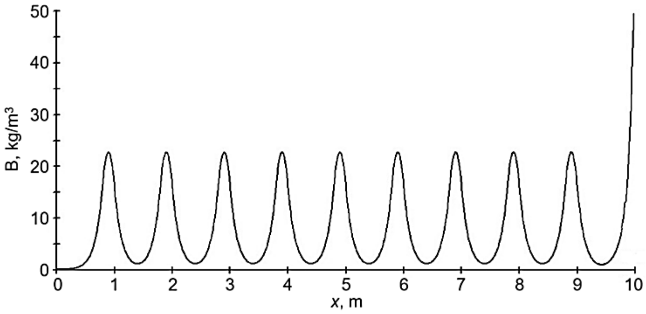

Figure 2 – Biomass distribution at t = 720 days

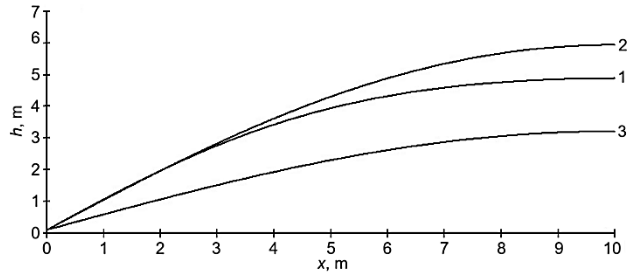

Figure 3 – Distribution of excess pressure for the case of considering bioprocesses: 1 – 60 days; 2 – 170 days; 3 – 250 days

5 Discussion

According to the classical theory of filtration consolidation of porous media, the applied external loads or the self-weight of the solid phase of the medium causes excess pressures in the pore fluid. From the perspective of porous media mechanics, excess pressures are considered detrimental, as they can lead to loss of medium stability and failure.

As these pressures dissipate (the dynamics of pressure changes are described by equation (1)), the medium undergoes compaction, reduction in pore volume, and, consequently, surface subsidence. Therefore, according to the classical consolidation theory, there is an evident interdependence: decreasing excess pressures leads to more significant subsidence and vice versa [33].

However, as the above results demonstrate, considering the development of microorganism biomass and its influence on pressures and subsidence runs directly counter to classical notions.

The effect of bioprocesses accelerates the subsidence of the surface of the porous medium, even though the maximum values of excess pressure remain somewhat larger (Table 1). The development of biomass (Figure 2) increases pressure in the pore fluid because the porosity decreases.

As a result, the maximum pressure values consistently exceed those for neglecting bioprocesses when considering bioprocesses. However, additional pressures increase the filtration rate and the volume of the filtered liquid due to the reduction of the pore space despite some reduction in the filtration coefficient from the Kozeny–Carman equation. Therefore, according to condition (13), this leads to an increase in subsidence.

The study of consolidation processes, surface subsidence, and dissipation of excess pressures is essential for assessing the stability of slopes in solid household waste landfills [34].

Predicting and accounting for factors deviating from the classical consolidation problem is critical in accurately assessing stability coefficients of porous medium masses, such as waste landfills.

6 Conclusions

A mathematical model of subsidence of a porous medium based on the theory of filtration consolidation that considers the effect of microorganisms was constructed and investigated. Obtained results of numerical experiments seem somewhat paradoxical at first glance. When considering the effect of the development of microorganisms, the excess pressures remain consistently more significant, as well as the amount of subsidence of the surface of the porous medium.

However, according to the classical theory, slowing the dissipation of excess pressures means slowing subsidence. The reasons for the obtained results of the investigated non-classical problem were explained based on the effect of the formation of additional biomass.

Overall, the research represents the initial, essential stage towards comprehensively considering the interrelated processes of filtration consolidation, subsidence, and reproduction of microorganism biomass concerning waste landfills.

Further research will involve accounting for processes of greenhouse gas generation as a result of microorganism activity in environments with organic components.

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